SAFE-DISPERSION: A graphical user interface for modeling guided wave propagation in elastic solids

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ABSTRACT

This paper presents a graphical user interface (GUI) for modeling ultrasonic guided wave propagation in elastic solids. The software exploits the semi-analytical finite element (SAFE) method for the calculation of wave-propagation characteristics. The interface allows for the modeling of piezoelectric effects in plate-like and arbitrary cross-sectional waveguides. The isotropic and anisotropic materials with damping effects are also considered. For anisotropic composite material cases, directivity plots can be extracted, containing the phase-velocities, group velocities, and slowness curves. The frequency-dependent mode shapes can also be obtained, including displacement, strain, stress, and other electric components for piezoelectric materials. The corresponding mode shapes for arbitrary cross-sectional waveguides are presented in the form of vivid animations, demonstrating the cross-sectional harmonic motions. All the computational outcomes are compared with commercial finite element (FE) codes via the Bloch-Floquet boundary conditions. The paper finishes with discussion, concluding remarks, and suggestions for future work.

Keywords: graphical user interface, guided waves, piezoelectric materials, dispersion, semi-analytical finite element method, waveguides

1. INTRODUCTION

Ultrasonic guided waves have been widely investigated as a powerful tool for structural sensing applications to establish self-awareness of smart structures. The peculiar multimodal and dispersive wave characteristics vary among different structural materials and waveguide cross-sections. The accurate computation of dispersion curves plays an essential role for the effective design of elastic wave-based Structural Health Monitoring (SHM) systems[1-3].

The computational transfer matrix formulations for multilayered structures have been developed[4], giving birth to the commercially available software DISPERSE™. The semi-analytical finite element (SAFE) method has also been widely investigated for modeling wave propagation in viscoelastic structures with arbitrary cross sections[5, 6]. Joseph proposed a hybrid SAFE-GMM (global matrix method) to obtain the stress mode shapes in layered media[7]. In particular, the SAFE technique was further extended for dispersion curves calculation in piezoelectric materials[8-10]. Kalkowski et al. employed the SAFE method to model the piezoelectric excitation[11]. In addition, the dispersion curves of arbitrary waveguides can also be achieved using Bloch-Floquet theorem finite element (FE) method[12, 13]. Based on these pioneer research activities, this study aims at developing a versatile software for modeling wave propagation in smart materials and structures with arbitrary cross sections.

It can be noted that the existing software for the calculation of dispersion curves mainly focus on the regular cross-sectional waveguides. On the other hand, the graphical user interface (GUI) which incorporates the piezoelectric effects for complex waveguides is rare. This research strives to establish a new, versatile, and user-friendly GUI for wave dispersion characteristics computation in arbitrary cross-section waveguides. Furthermore, compared with the FE simulation approach, the superiority of the designed software resides in its capability of rendering the complex valued frequency-wavenumber relations for capturing the attenuated propagative waves and the evanescent waves.

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This research initiates with the coupled field SAFE formulations. The semi-analytical approach was coded into MATLAB and the SAFE-DISPERSION GUI was developed. The user-friendly interface allows for the selection of piezoelectric effects in plate-like, cylindrical, and arbitrary cross-sectional waveguides. Finally, three case studies regarding different materials and structures are conducted: rails, piezoelectric plates, and piezoelectric pipes. The comparison between the SAFE solutions and the commercial FE code (ANSYS) results are presented to showcase the accuracy in wave dispersive characteristics computation.

2. COUPLED FIELD SAFE FORMULATIONS

The classic SAFE formulations can be found in many existing literatures[5, 8, 9]. The proposed software exploits the SAFE expressions and facilitates the post-processing module as well. Here, the coupled field SAFE solutions will be emphasized which is the core of modeling wave propagation in piezoelectric materials.

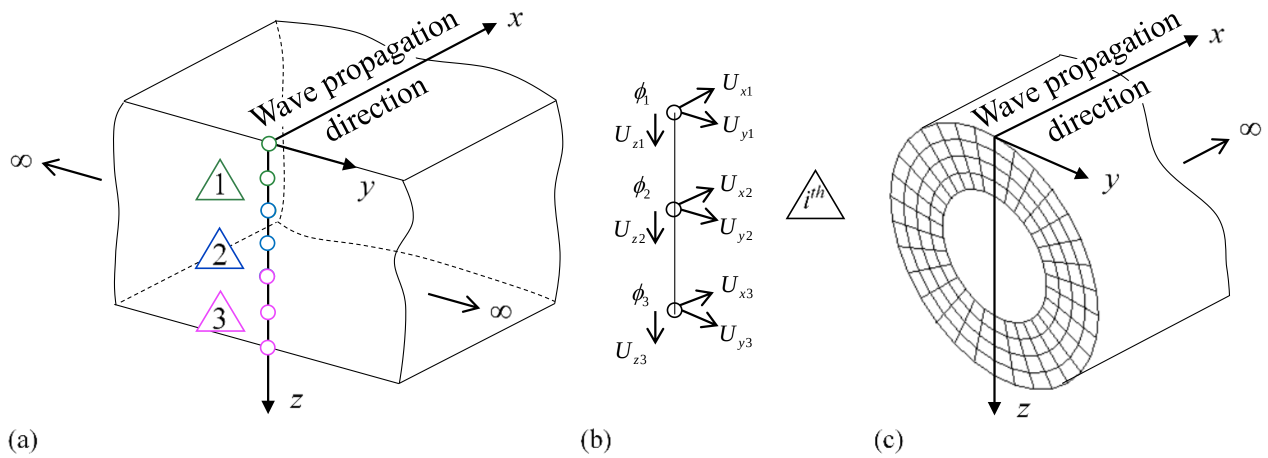

Figure 1. (a) one-dimensional SAFE model of wave propagation; (b) degrees of freedom of a one-dimensional three-node element with piezoelectric effects; (c) a 2D SAFE model of wave propagation in pipe structures.

Both one-dimensional and two-dimensional SAFE models are shown in Figure 1. It is assumed that the guided waves propagate along the *x* direction and the cross section lies in the *y*-*z* plane. For the 1D case, the piezoelectric plate was discretized in the thickness (*z*-axis) direction using three-node finite elements. For example, Figure 1(a) presents an infinite piezoelectric plate discretized by three elements along the thickness direction. Figure 1(b) displays the nodal degrees of freedom in one element, including the displacements in all the three directions (U_x, U_y, U_z) and the electric potential (ϕ). Figure 1(c) illustrates a 2D SAFE model whose cross section is discretized by four-node finite elements. The constitutive equation for piezoelectric materials can be expressed as[14]

$$Q = C^* q \tag{1}$$

where Q is a vector of the stress components and the electric displacements; q denotes the vector with components of the mechanical strain and electric field; C^* represents a matrix containing elastic constants c, piezoelectric stress constants e, and dielectric constants ε:

$$C^* = \begin{bmatrix} c & -e^T \\ e & \varepsilon \end{bmatrix} \tag{2}$$

Considering the elemental shape functions, the equation of motion can be transformed into

$$(K_1 - ikK_2 + k^2K_3 - \omega^2 M)V = 0 \quad (3)$$

where K_1 , K_2 , K_3 are element assembly stiffness matrices; M is element assembly mass matrix; k denotes the wavenumber along x direction; ω represents the angular frequency; V stands for the nodal degrees of freedom. The dispersion relationships can be achieved by solving the standard eigenvalue problem. In this study, the complex-valued roots (wavenumber k) are calculated for the given angular frequency ω . Therefore, both evanescent waves and attenuated propagative wave modes can be revealed through the proposed approach. In addition, to better excavate the wave dispersive characteristics in piezoelectric composite plates, the group velocities adopt the formulation of circular-crested guided waves in anisotropic laminate composites[15].

3. SAFE-DISPERSION INTERFACE AND MAIN FUNCTIONS

The semi-analytical representations of SAFE formulations were coded with MATLAB and resulted in the GUI called the SAFE-DISPERSION. The main interface is shown in Figure 2. SAFE-DISPERSION allows users to select the waveguide cross-section: 1D (infinite plates), 2D classic shapes (beams or cylinders), and 2D generic shapes (rails, T-shape bars, etc.). Furthermore, the piezoelectric effects can also be chosen to be activated or deactivated. It means that the designed software enables the computation of wave dispersion characteristics for arbitrary cross sections with piezoelectric effects.

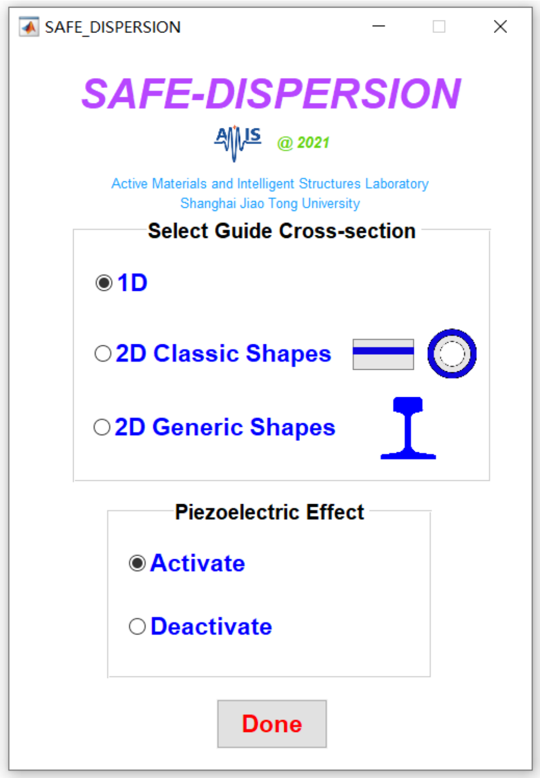

Figure 2. Main GUI of SAFE-DISPERSION.

This study focuses on the investigation of guided wave dynamics in piezoelectric materials. Therefore, the GUI interface regarding the piezoelectric module will be presented in detail. Figure 3 illustrates the user interface for 1D piezoelectric case. Users need to define the total number of structural layers. For each layer, thickness, the number of elements for discretization, and stacking angle can be optionally defined. Besides, typical piezoelectric material properties are provided for the users' convenience. It should be noted that the material properties for each layer are dependent on the stacking angle, which strictly follows the coordinate transformation principle[16]. In addition to inputting the piezoelectric parameters, users need to set the frequency range and sweeping step. In particular, the 1D case is able to compute the wave dispersive features in all the directions, expressing the directivity plots of phase velocities, group velocities, and slowness curves. All the calculated results are fully available to the user and could be saved by clicking the “SAVE” button.

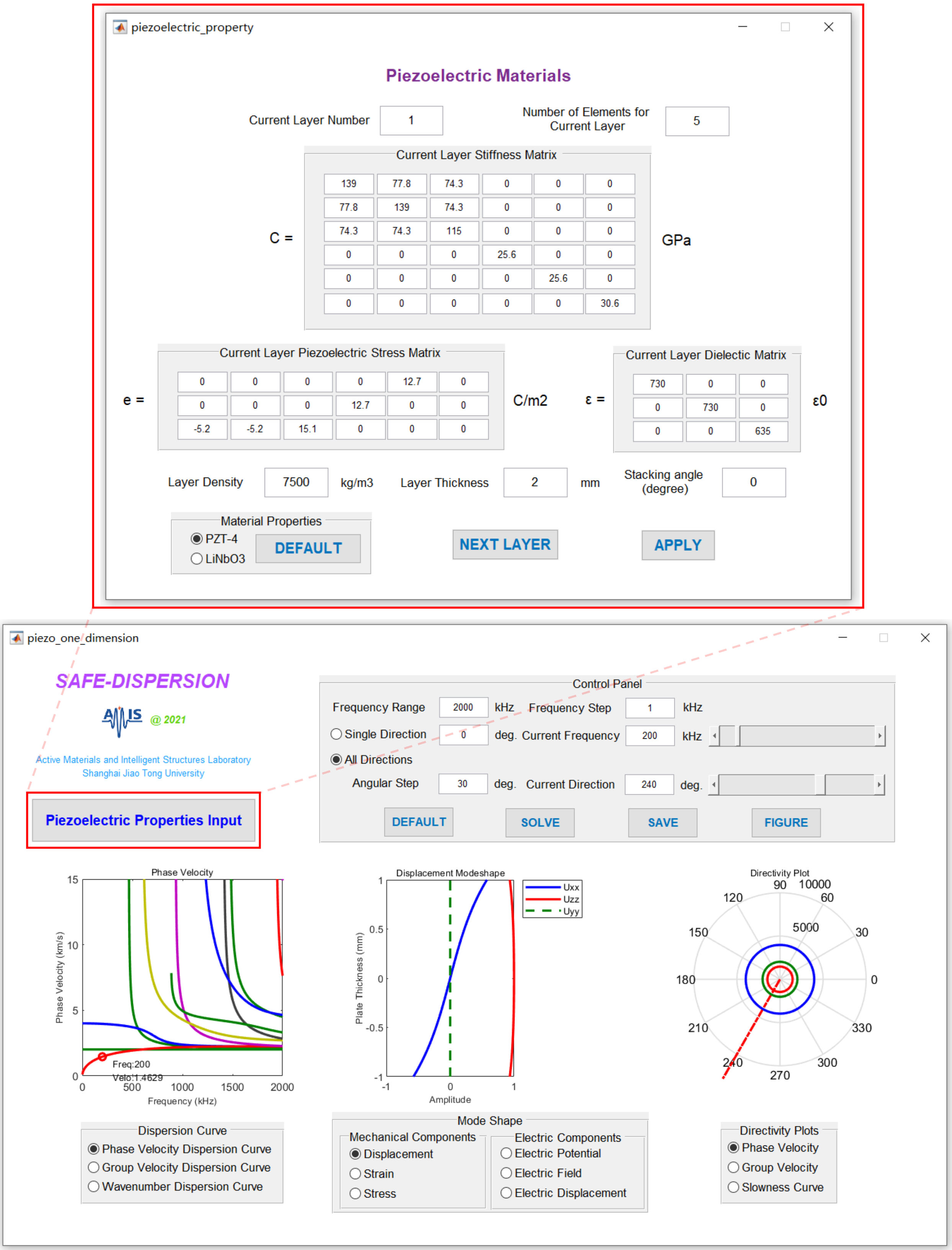

Figure 3. User interface for 1D piezoelectric case.

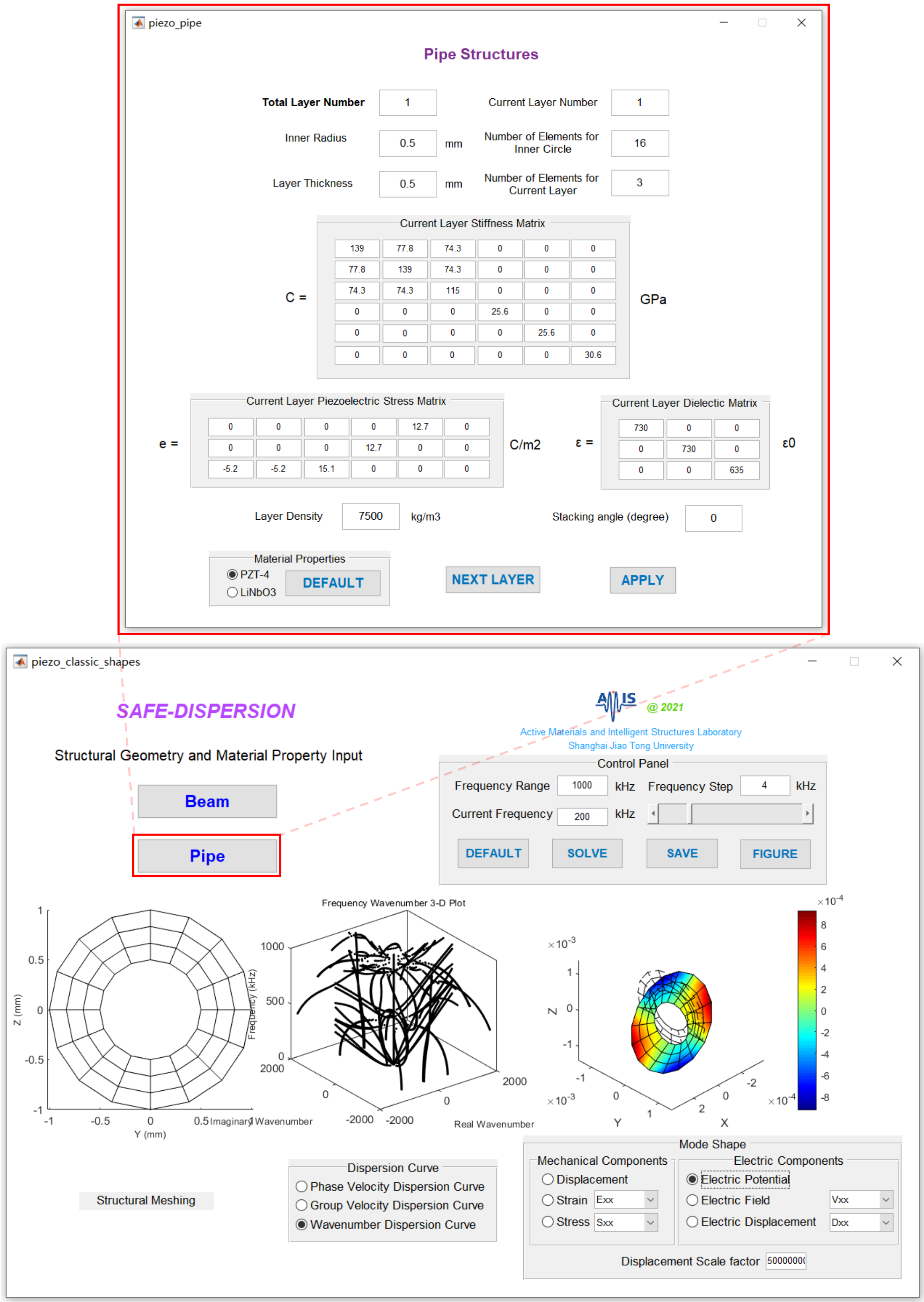

Figure 4. User interface for a 2D classical shape case with piezoelectric effects.

Figure 4 presents the user interface for a 2D classical shape case with piezoelectric effects. Similar to the 1D case, the total number of layers can be defined as well as the elements for each layer. The structural geometry and meshing are plotted in the main interface after clicking the “APPLY” button. The mechanical and electric mode shape components are displayed in the form of vivid animation, rendering the harmonic motion patterns. For comparison, the undeformed configuration is also retained within the animation. The users can click on the frequency-phase velocity curves to look up another frequency-dependent mode.

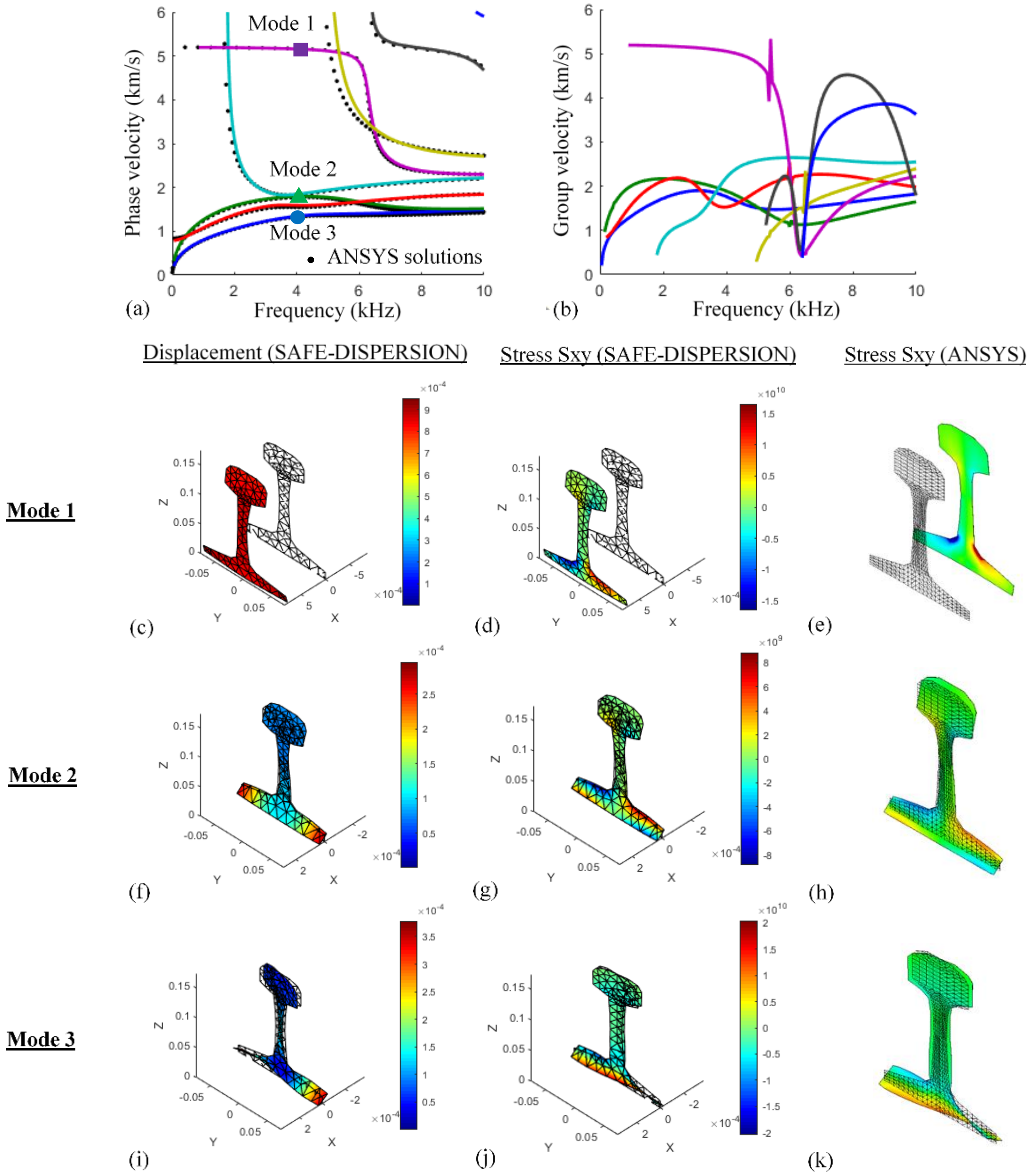

Figure 5. Dispersion characteristic results of a free P60 rail: (a) phase velocity dispersion curves; (b) group velocity dispersion curves; (c) displacement mode shape for mode 1; (d) stress mode shape for mode 1; (e) stress mode shape for mode 1 from finite element simulation; (f) displacement mode shape for mode 2; (g) stress mode shape for mode 2; (h) stress mode shape for mode 2 from finite element simulation; (i) displacement mode shape for mode 3; (j) stress mode shape for mode 3; (k) stress mode shape for mode 3 from finite element simulation.

4. CASE STUDIES

To comprehensively show the features of the proposed software, several case studies were conducted, modeling guided wave propagation in a free rail, a piezoelectric plate, and a piezoelectric pipe. All the computational results are compared with the outcomes obtained by the commercial finite element code (ANSYS) with Bloch-Floquet boundary conditions.

4.1 Dispersive waves in a free rail

The multimodal dispersive waves in a free rail have been widely studied through numerical simulations and experimental demonstrations[5, 17]. Compared with the previous investigations, the developed GUI interface further captures the mechanical strain and stress information of guided wave modes in the rail. Figure 5 presents the dispersion results of a typical P60 steel rail. It can be found that the phase velocities attained by the SAFE formulations match well with the ANSYS solutions. In addition, the corresponding group velocities can also be obtained. It should be noted that more discretized elements should be implemented to meet the accuracy requirement with increasing cost of computational time and resource.

Three typical modes at 4 kHz were chosen, marked by different colors and shapes in Figure 5(a). The displacement and stress component S_{xy} mode shapes from the proposed software and commercial finite element code were plotted and compared. The first mode represented the fundamental symmetric mode, as the displacements were symmetric with respect to the *x*-*z* plane. Furthermore, the extrema of stress S_{xy} were located on the surfaces of the rail foot, which were captured equally well through the proposed software and the finite element simulation. The displacements of the second mode were antisymmetric with regard to the *x*-*y* plane, while the rail foot presented the symmetric displacements with reference to the *x*-*z* plane. The third mode exhibited the fundamental antisymmetric mode, i.e., the displacements were antisymmetric with respect to both *x*-*z* and *x*-*y* plane. In particular, the maximal stress S_{xy} can be noticed on the bottom surface of the rail foot. It can be noted that some of the modes can only vibrate a certain region of the rail, such as the rail head and foot. Therefore, it is necessary to know the sensitive wave modes to various locations in the rail cross-section. This aspect may help design the effective actuation and sensing system in practical Non-destructive Evaluation (NDE) and SHM applications.

4.2 Guided wave propagation in a piezoelectric plate

The piezoelectric module is the fundamental portion of the developed software. The current case study initiates with the 1D case, fully investigating the wave dispersive characteristics in piezoelectric materials. For the finite element simulation approach, the procedure to obtain the dispersion curves in piezoelectric plates can be found in Ref. 18.

Figure 6 presents the dispersion results of a 2-mm thickness piezoelectric plate. The 3D complex-valued frequency-wavenumber relationships are shown in Figure 6(a). It can be observed that the dispersion curves unveil stagger and continuous traits, which covers both evanescent and attenuated propagative wave modes. Here, the wave velocities and mode shapes of the propagative modes will be stressed. According to the phase velocity plots, the computational results superpose with the ANSYS solutions below 1500 kHz. However, they deviate a bit from each other within the high frequency range. The reason lies in that when it approaches to high frequency, the wavelengths become shorter. Therefore, more elements are required to discretize the plate along the thickness direction for the purpose of capturing the accurate wave mode features. The corresponding group velocities are also plotted in Figure 6(c).

The fundamental symmetric and antisymmetric wave modes, S0 and A0 at 200 kHz, were selected to present the mechanical and electric mode shapes. For S0 mode, the displacements in the wave propagation direction were symmetric along the thickness direction, while the displacement in the thickness direction and the electric potential kept antisymmetric. This aspect was opposite to the A0 mode, i.e., the electric potential presented the symmetric pattern along the thickness direction. Furthermore, both displacement and electric potential mode shapes obtained from the developed software and commercial finite element code results rendered perfect superposition. Considering the electric displacement mode shapes, the component in the wave propagation direction of S0 mode was antisymmetric with respect to the thickness direction, while A0 mode demonstrated the symmetric feature. It can be found that these distinctive wave features may influence the wave excitation and sensing capabilities in piezoelectric materials.

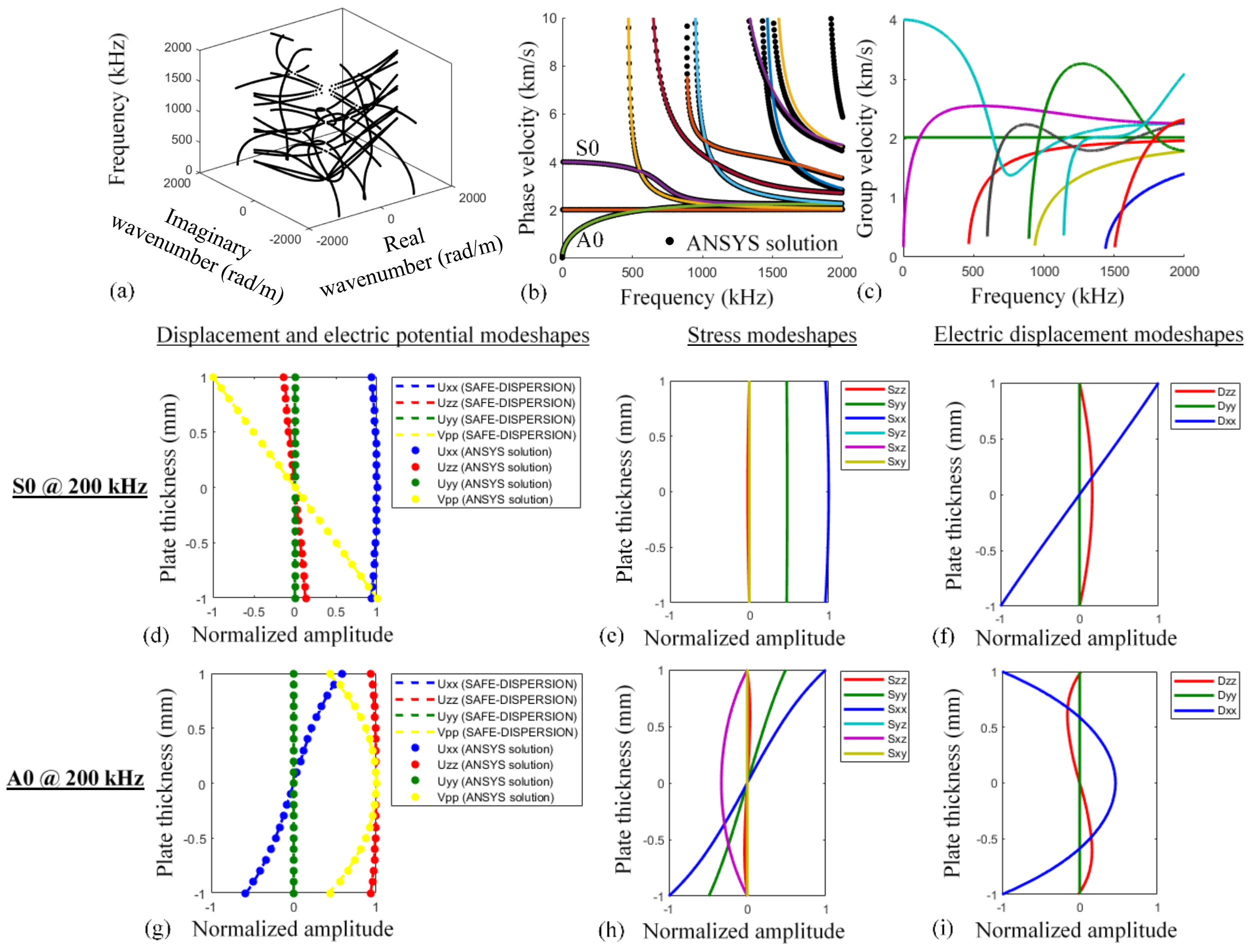

Figure 6. Dispersion characteristic results of a 2-mm thick piezoelectric plate: (a) complex wavenumber-frequency relationship; (b) phase velocity curves; (c) group velocity curves; (d) displacement and electric potential mode shapes for S0 mode at 200 kHz; (e) stress mode shapes for S0 mode at 200 kHz; (f) electric displacement mode shapes for S0 mode at 200 kHz; (g) displacement and electric potential mode shapes for A0 mode at 200 kHz; (h) stress mode shapes for A0 mode at 200 kHz; (i) electric displacement mode shapes for A0 mode at 200 kHz.

4.3 Guided wave propagation in a piezoelectric pipe

The overall understanding of the coupled field SAFE formulations was extended to the 2D case. Here, a piezoelectric pipe poled along the *z* direction was investigated as a case study. The inner diameter was 1 mm, and the external diameter was 2 mm. Figure 8 shows the dispersion results of the piezoelectric pipe. The 3D complex frequency-wavenumber figure well captures the evanescent and attenuated propagative modes. In terms of the phase velocities, the deviation between the computational results and finite element solutions took place above 400 kHz. Compared with the 1D case, the two-dimensional cross-section needs more elements to meet the computational precision. Consequently, it may result in a relatively large computational burden. The group velocities are also plotted in Figure 8(c).

Three fundamental wave modes, L0, T0, and F0 at 200 kHz, were chosen as the examples for illustrating the displacement and electric potential mode shapes. The color map in Figure 7 presents the value of electric potential. In general, both SAFE-DISPERSION and finite element simulation well captured the distinct wave dynamic characteristics. Besides, the electric potential of L0 and T0 modes shows that the extrema of electric potential locate on opposite sides. However, for F0 mode, equivalent electric potential was located at opposite sides. It should be noted that the polarization of the piezoelectric pipe in this study was along the *z* direction, while the available polarization of a piezoelectric cylinder should always trace along the thickness direction. Therefore, the SAFE formulations to realize this aspect is still worth of investigation and will be conducted in a future work.

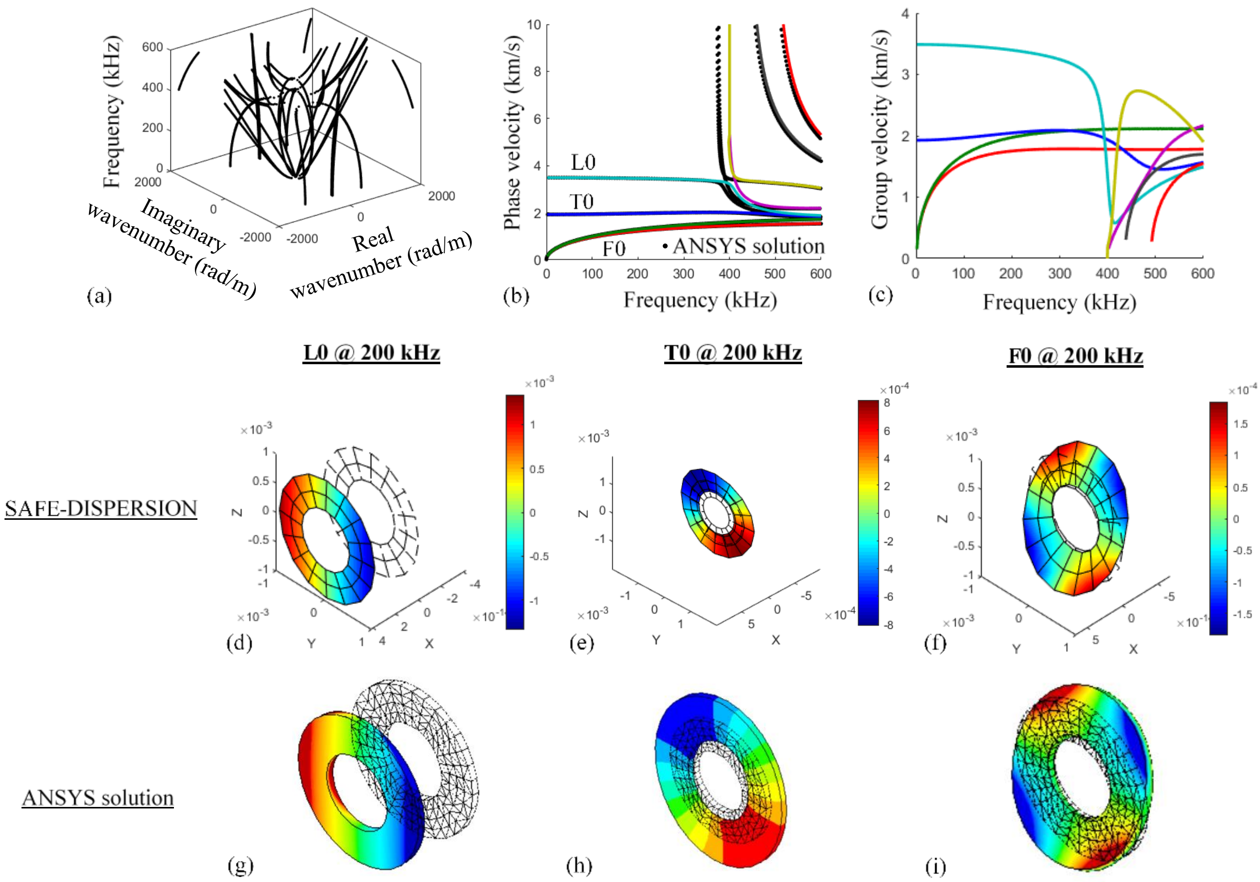

Figure 8. Dispersion characteristic results of a piezoelectric pipe: (a) complex wavenumber-frequency relationship; (b) phase velocity curves; (c) group velocity curves; (d) displacement and electric potential mode shapes for L0 mode at 200 kHz via SAFE-DISPERSION; (e) displacement and electric potential mode shapes for T0 mode at 200 kHz via SAFE-DISPERSION; (f) displacement and electric potential mode shapes for F0 mode at 200 kHz via SAFE-DISPERSION; (g) displacement and electric potential mode shapes for L0 mode at 200 kHz via ANSYS solution; (h) displacement and electric potential mode shapes for T0 mode at 200 kHz via ANSYS solution; (i) displacement and electric potential mode shapes for F0 mode at 200 kHz via ANSYS solution.

5. CONCLUDING REMARKS AND FUTURE WORK

This paper presented a graphical user interface for modeling ultrasonic guided wave propagation in elastic solids. One dimensional coupled field SAFE formulations were illustrated, and then extended to arbitrary cross-sectional waveguides. The new and versatile software was designed for wave dispersion calculations which integrated the piezoelectric effects for complex waveguides potentially used in smart structures. Case study examples were conducted, modeling guided wave propagation in a free rail, a piezoelectric plate, and a piezoelectric pipe. The corresponding dispersion curves and mode shapes demonstrated the outstanding agreement between the proposed software and commercial finite element codes.

For future work, the SAFE formulations for piezoelectric cylinders poled across the thickness direction should be investigated. And examples of the generic shape cases with piezoelectric effects should also be conducted.

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