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A SPECTRAL CORRELATION BASED NONLINEAR ULTRASONIC RESONANCE TECHNIQUE FOR FATIGUE CRACK DETECTION

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ABSTRACT

This paper presents a spectral correlation based nonlinear ultrasonic resonance technique for fatigue crack detection. A reduced-order nonlinear oscillator model is initially constructed to illuminate the Contact Acoustic Nonlinearity (CAN) and nonlinear resonance phenomenon. The tailored analytical model considers the rough surface condition of the fatigue cracks, with a crack open-close transition range for the effective modeling of the variable-stiffness CAN. Multiple damage indices (DIs) associated with the degree of nonlinearity of the interrogated materials are then proposed by correlating the ultrasonic resonance spectra. The frequency sweeping signals serve as the excitation waveform to obtain the structural dynamic features. The nonlinear resonance procedure is numerically solved using the central difference method. Short time Fourier transform (STFT) is utilized to extract the resonance spectroscopy. In this study, pristine, linear wave damage interaction case (an open notch case), and nonlinear wave damage interaction case (a fatigue crack case) with various damage severities are considered. Subsequently, three case studies taking advantage of different nonlinear oscillation phenomena are conducted based on the spectral correlation algorithm to detect and monitor the fatigue crack growth: time-history dependence, amplitude dependence, and breakage of superposition. Each of these three nonlinear behaviors can either work individually or collaborate synthetically to detect the nucleation and growth of the fatigue cracks. The proposed nonlinear ultrasonic resonance technique possesses great application potential for fatigue crack detection and quantification. This paper finishes with summary, concluding remarks, and suggestions for future work.

Keywords: nonlinear ultrasonics, fatigue crack detection, spectral correlation, contact acoustic nonlinearity, structural health monitoring, nondestructive evaluation

1. INTRODUCTION

Fatigue cracks can be widely found in structural components under cyclic loadings. They are barely visible and hard to detect, serving as a major cause of failures in metallic structures. Ultrasonic techniques have been found to be promising for fatigue crack detection among the Structural Health Monitoring (SHM) and Nondestructive Evaluation (NDE) communities. The linear ultrasonic features are not sensitive enough to detect fatigue cracks until they become visibly large. However, the nonlinear counterpart proves to be more sensitive to such incipient changes as fatigue cracks [1]. Extensive efforts have been exerted on the investigation of several distinctive nonlinear features, such as sub/super harmonic generation, DC response, mixed frequency modulation, and nonlinear resonances [2]. As a consequence, nonlinear guided wave based inspection techniques have shown great potential for fatigue crack detection and monitoring.

Many researchers have conducted the investigation on the nonlinear vibro-acoustic techniques for damage detection. Klepka et al. experimentally investigated the vibro-acoustic nonlinear wave modulations, where they found that wave modulation intensity strongly depended on fatigue crack modes [3]. Solodov presented the study to enhance the sensitivity of nonlinear NDE using the local defect resonance [4]. The Nonlinear Elastic Wave Spectroscopy (NEWS) methods were adopted to detect delamination damage due to low velocity impact [5] The spectral correlation algorithm has been widely utilized for fatigue crack detection combining with other ultrasonic inspection techniques [6]. All these investigations have demonstrated the superior sensitivity and damage detection capability of nonlinear ultrasonic techniques for SHM and NDE.

In addition to the aforementioned experimental studies, extensive modeling and simulation efforts have been exerted on understanding and predicting the nonlinear resonance phenomenon. Broda et al. presented different types of elastic and

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dissipative models of nonlinear wave crack interactions related to structural damage detection [7]. Both analytical approach and finite element simulation were achieved for modeling nonlinear guided wave propagation [8, 9]. Shen and Cesnik extended the Local Interaction Simulation Approach (LISA) to efficiently model the nonlinear interactions between ultrasonic guided waves and fatigue cracks [10]. Radecki et al. employed a spring model in the LISA formulation and simulated the nonlinear Lamb wave propagation at the breathing cracks [11]. These predictive models provide powerful tools for the analysis of nonlinear ultrasonic phenomena behind the SHM and NDE methods.

This study starts with the presentation of a nonlinear oscillator model for the analysis of ultrasonic resonance phenomena by introducing a transitional region of a rough crack interface with initial openings and closures. Subsequently, the concept of multiple Damage Indices (DI) is illustrated, defined by correlating the ultrasonic spectra. Three case studies are carried out using the spectral correlation algorithm for fatigue crack detection based on time-history dependence, amplitude dependence, and breakage of superposition. The individual and fused DIs show the superb capability to detect the nucleation and growth of the fatigue cracks.

2. A REDUCED-ORDER NONLINEAR OSCILLATOR MODEL FOR ROUGH FATIGUE CRACK DYNAMICS

The bilinear models for nonlinear wave crack interactions have been widely investigated for the simulation of idealized breathing cracks [3]. However, the practical fatigue crack conditions may give rise to totally different contact behavior. According to our previous investigation [12], the microscopic images of fatigue cracks generally display rough interfaces with zigzagged crack traces and randomly distributed initial openings and closures as shown in FIGURE 1a. Based on these observations, this study proposes a nonlinear oscillator model with a transitional region to capture the rough crack surface features.

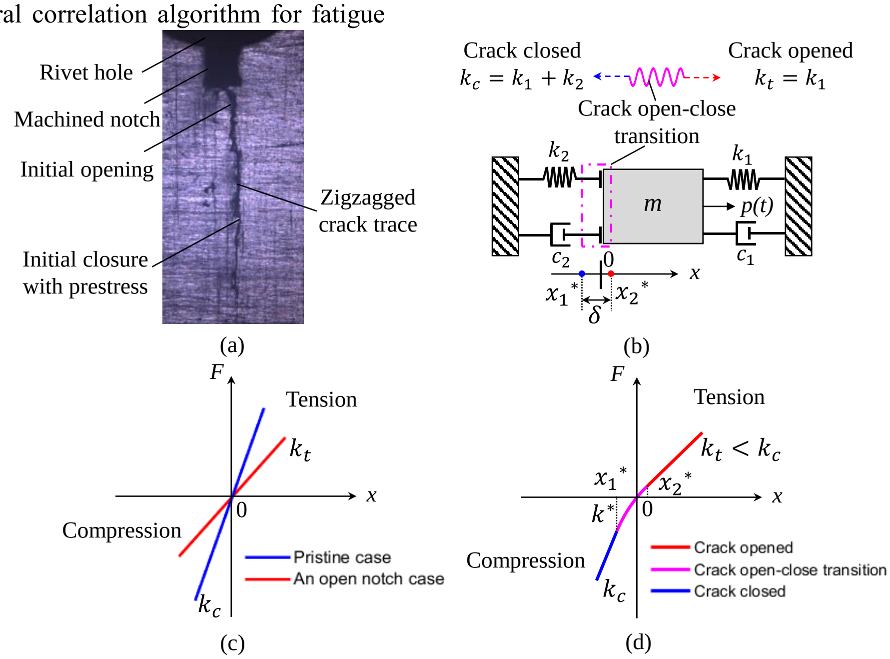

FIGURE 1: (A) MICROSCOPIC IMAGE OF A RIVET HOLE NUCLEATED FATIGUE CRACK; (B) REDUCED-ORDER NONLINEAR OSCILLATOR MODEL FOR CAN CONSIDERING A TRANSITIONAL REGION FOR CAPTURING THE ROUGH CRACK CONDITION; (C) LINEAR FORCE AND DISPLACEMENT RELATIONSHIP FOR A PRISTINE AND AN OPEN NOTCH CASE; (D) NONLINEAR SPRING FORCE DISPLACEMENT RELATIONSHIP FOR A CRACKED BODY.

FIGURE 1b shows the reduced-order nonlinear oscillator model for the simulation of the CAN considering a transitional region for capturing the rough crack condition. The pristine and linear wave damage interaction cases are also investigated. FIGURE 1c presents the relationship between force and displacement of a pristine and an open notch case. FIGURE 1d illustrates the nonlinear spring force and displacement for a cracked body. The entire oscillation procedure undergoes three stages: crack open, crack open-close transition, and crack close. The subscripts C and T denote "compression" and "tension" for status of the fatigue crack. The parameters x_1^* and δ represent the completely closed and completely opened crack surface displacements. The vibration of such nonlinear oscillator is governed by the classical nonlinear dynamic equation, i.e.,

$$m\ddot{u}(t) + c(u)\dot{u}(t) + k(u)u(t) = p(t) \tag{1}$$

where m is the mass; $u(t)$ represents the time dependent displacement; $c(u)$ refers to the displacement dependent damping coefficients; $k(u)$ depicts the apparent stiffness, varying between the compressional and tensile status; $p(t)$ means the transient external excitation.

A transitional region δ stemming from the roughness of the crack interface is introduced into the analytical formulation, yielding

$$\delta = \phi \cdot \delta + (1-\phi) \cdot \delta = -x_1^* + x_2^* \tag{2}$$

where ϕ is the ratio between the crack completely-closed stroke displacement and the transitional dimension. When the crack is under tension and completely opened ($x > x_2^*$), the structural stiffness takes the value of $k_t = k_1$; while when the crack is under compression and completely closed ($x < x_1^*$), the structural stiffness increases to $k_c = k_1 + k_2$; as for the crack open-close transitional region, the structural stiffness varies, following a polynomial curve joining the completely open and close status. In the pristine case, the structural stiffness keeps constant under both compression and tension, taking the value of k_c. On the other hand, the stiffness for the linear wave damage interaction case (such as an open notch) is k_t.

To investigate the nonlinear resonance phenomenon, a dynamic system with a natural frequency of 200 kHz was considered as a case study. Under compression, the structure behaves like a continuum, so $\omega_c = \omega_n$ which is the natural frequency of the structural component in the pristine condition. Under tension, the crack completely opens, the structural component becomes discontinuous, resulting in a decreased stiffness. Hence, it is obvious that $\omega_t < \omega_n$. ω_t directly relates to the severity of the damage in the structure. A relative crack size r is introduced. It is assumed that $\omega_t = \omega_n \sqrt{1-r}$. The crack open-close transitional stage takes an intermediate natural frequency depending on the vibration amplitude. The damping ratio ξ can be evaluated by

$$c(u) = 2\omega(u) m \xi \tag{3}$$

The nonlinear vibration Eq. (1) can be readily solved numerically using the classical central different method with a time marching procedure. The details of such numerical implementation and discussion can be found in an extended journal report by the authors. This paper does not intend to include all the step-by-step derivations. It aims at communicating our key results with the SHM and NDE community.

3. SPECTRAL CORRELATION BASED DAMAGE INDEX

The spectral correlation method has been manifested as a powerful tool for analyzing cyclo-stationary vibration signals. In this study, the spectral correlation algorithm was utilized for establishing Damage indices (DI) based on the nonlinear resonance phenomena. Given a time domain resonant response $x(t)$, its spectral density function can be evaluated by

$$D(f) = E[X(f) X^*(f)] \tag{4}$$

where f stands for the target frequency range; $X(f)$ is the Fourier transform of $x(t)$; * denotes the complex conjugate; E represents the expectation operation. In this study, the spectral density directly reflects the structural response in the frequency domain. Three case studies demonstrating different aspects of nonlinear resonance features are carried out, i.e., time-history dependence, amplitude dependence, and breakage of superposition. In each case study, two resonant signals need to be processed by the spectral correlation algorithm. Therefore, the deviation between them gives rise to the DI in the form of

$$DI = 1 - \frac{\left| \int_{f_1}^{f_2} D_1(f) D_2(f) df \right|}{\sqrt{\int_{f_1}^{f_2} D_1^2(f) df \int_{f_1}^{f_2} D_2^2(f) df}} \tag{5}$$

where $D_1(f)$ and $D_2(f)$ represent the spectral density functions of individual resonant response in each measurement case; f_1 and f_2 denote the frequency extrema in the target frequency band.

The procedure to apply the spectral correlation based nonlinear ultrasonic resonance method can be achieved in the following steps:

STEP 1: Apply the excitation signals of each case study on the nonlinear oscillator model and obtain the corresponding temporal resonant responses given by Eq. (1).

STEP 2: Normalize the time trace signals. It should be noted that for the breakage of superposition case study, the two resonant responses under lower amplitude excitation should be superposed before the signal normalization operation.

STEP 3: Evaluate the spectral density of each normalized response according to Eq. (4).

STEP 4: Calculate the respective DI of each case study in Eq. (5).

4. ANALYTICAL CASE STUDIES FOR NONLINEAR RESONANCE SPECTROSCOPY

This section utilized the reduced-order rough crack model to demonstrate special nonlinear resonance features that may allow us to detect and quantify fatigue cracks. A series of case study results will be presented. In this study, three distinctive nonlinear features are investigated for fatigue crack detection via the resonance approach, i.e., time-history dependence, amplitude dependence, and breakage of superposition. These features facilitates the development of the proposed fatigue crack detection technique.

4.1 Excitation Signal

In order to obtain a comprehensive information about the nonlinear dynamic features of a structure, the frequency sweeping excitation is usually adopted [13]. FIGURE 2 presents the excitation signal in both time and frequency domain. The structure was driven by a Tukey window modulated chirp signal under a sweeping frequency from 50 kHz to 300 kHz. Both low frequency to high frequency and high frequency to low frequency excitation signals are used for the time-history dependence case study. Excitation signals with different amplitudes were exploited for both amplitude dependence and breakage of superposition case studies as well.

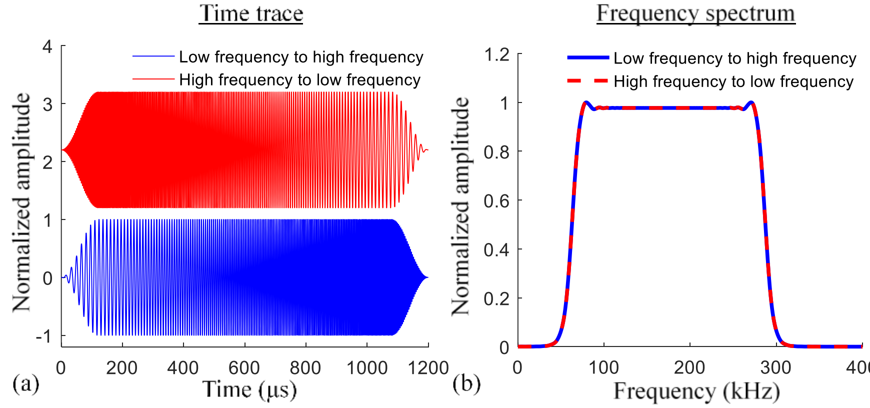

FIGURE 2: EXCITATION SIGNAL IN BOTH TIME AND FREQUENCY DOMAIN: (A) TIME TRACE; (B) FREQUENCY SPECTRUM.

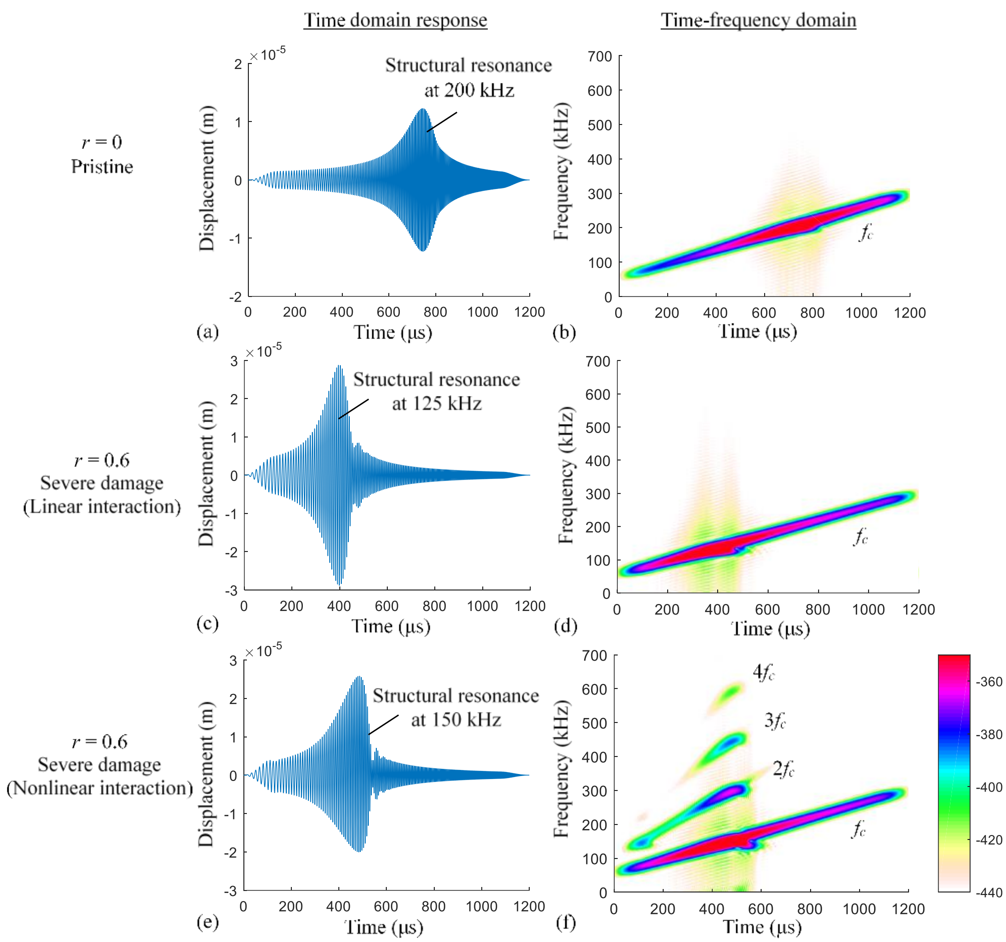

FIGURE 3: RESONANT RESPONSES OF THE PRISTINE, LINEAR NOTCH, AND NONLINEAR FATIGUE CRACK CASES: (A) TEMPORAL SIGNAL OF THE PRISTINE CASE; (B) SPECTROGRAM OF THE PRISTINE CASE; (C) TEMPORAL SIGNAL OF THE LINEAR INTERACTION CASE; (D) SPECTROGRAM OF THE LINEAR INTERACTION CASE; (E) TEMPORAL SIGNAL OF THE NONLINEAR INTERACTION CASE; (F) SPECTROGRAM OF THE NONLINEAR INTERACTION CASE.

4.2 Resonant Responses

In order to better illustrate the efficacy of the proposed nonlinear resonance technique, linear case (an open notch case), and nonlinear case (a fatigue crack case) are considered. FIGURE 3 shows the pristine, linear, and nonlinear ultrasonic resonant responses in the upward frequency sweeping direction. From the temporal responses, the shift of structural resonant frequency can be easily noticed. When it comes to the damaged cases, the resonant frequency decreases compared with the pristine case. In addition, the resonant frequency of the nonlinear interaction case is much higher than that of the linear case due to the variable stiffness during the nonlinear oscillation procedure. In order to extract the time-frequency features of structural response, short time Fourier transform (STFT) was performed on the vibration signals. In the pristine and linear interaction cases, only the fundamental excitation frequency can be observed. However, in the nonlinear interaction case, the second, third, and even fourth higher harmonic components appeared. Such a comparative case study demonstrates that our model can very well capture the nonlinear resonance phenomenon which embraces distinctive features for fatigue crack identification.

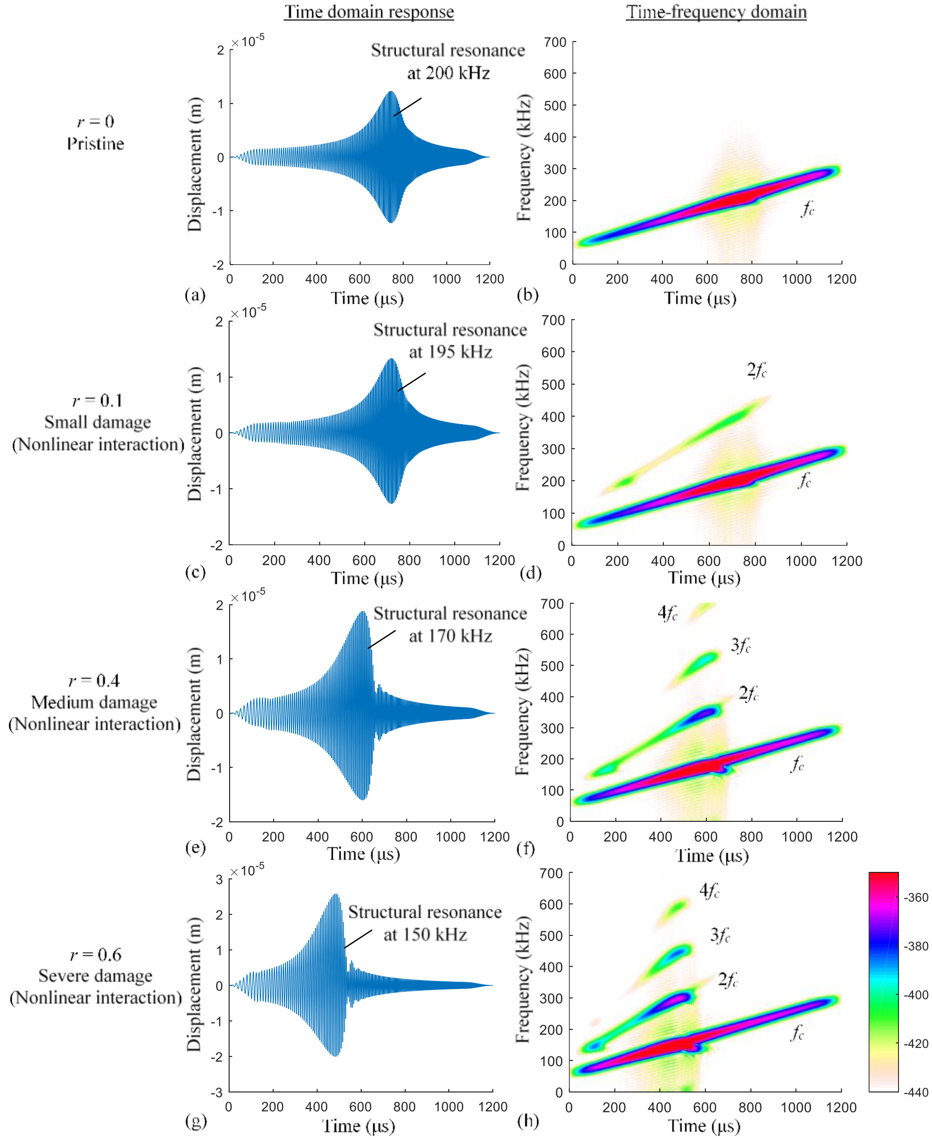

FIGURE 4: RESONANT RESPONSES OF THE PRISTINE AND NONLINEAR INTERACTION CASES: (A) TEMPORAL SIGNAL OF THE PRISTINE CASE; (B) SPECTROGRAM OF THE PRISTINE CASE; (C) TEMPORAL SIGNAL OF THE SMALL DAMAGE CASE; (D) SPECTROGRAM OF THE SMALL DAMAGE CASE; (E) TEMPORAL SIGNAL OF THE MEDIUM DAMAGE CASE; (F) SPECTROGRAM OF THE MEDIUM DAMAGE CASE; (G) TEMPORAL SIGNAL OF THE SEVERE DAMAGE CASE; (D) SPECTROGRAM OF THE SEVERE DAMAGE CASE.

In order to showcase the capability of nonlinear responses to monitor the nucleation and growth of fatigue cracks, both the pristine case and nonlinear case (a fatigue crack case) with different damage severities are studied. FIGURE 4 presents the resonance of the pristine structure as well as the nonlinear resonances for the three damage severity conditions. In this study, the damage size corresponds to a severe damage when the relative crack size r equals 0.6. From the temporal responses, it is remarkable that, with the increment of the damage severity, the resonant frequency decreases. In addition, only the second harmonic component exists in the small damage case (r =0.1). When it comes to the medium (r =0.4) and severe damage cases (r =0.6), the second, third, and even fourth higher harmonic components can be clearly noticed. It is apparent that the nonlinear features become more obvious with the damage severity increases. Therefore, the nonlinear ultrasonic resonance phenomena possess great potential for the fatigue damage detection and quantification.

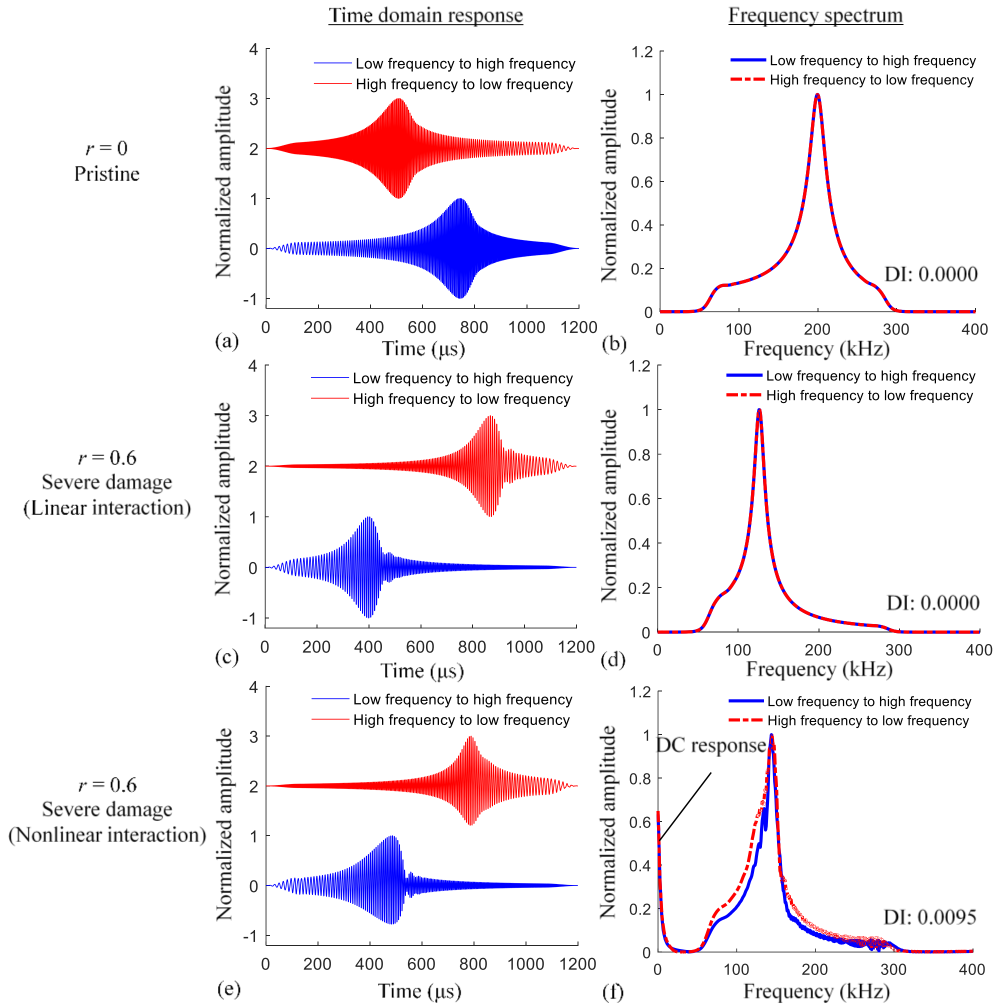

FIGURE 5: RESULTS OF TIME-HISTORY DEPENDENCE CASE STUDY: (A) TEMPORAL SIGNAL OF THE PRISTINE CASE; (B) FREQUENCY SPECTRUM OF THE PRISTINE CASE; (C) TIME TRACE OF THE LINEAR WAVE DAMAGE INTERACTION CASE; (D) FREQUENCY SPECTRUM OF THE LINEAR WAVE DAMAGE INTERACTION CASE; (E) RESPONSE SIGNAL OF THE NONLINEAR WAVE DAMAGE INTERACTION CASE; (F) FREQUENCY SPECTRUM OF THE NONLINEAR WAVE DAMAGE INTERACTION CASE.

4.3 Time-history Dependence

The evolvement of nonlinear response could highly depend on the loading history and the forcing sequence. This turned out to be quite evident for nonlinear resonance effect, giving rise to the special feature of time-history dependence. This study will show that as the excitation frequency is swept from different directions for the fatigue crack CAN, the response will evolve and develop in different behaviors, resulting in completely different responses curves. This case study strives to fully exploit the time-history dependence feature for fatigue crack detection.

The nonlinear oscillator model was excited by both upward and downward frequency sweeping signals with the same excitation amplitude. FIGURE 5 presents the results of the time-history dependence case study. In the pristine and linear wave damage interaction cases, the temporal responses for both upward and downward frequency sweeping directions showed a time-reversed relationship; the spectral components correspond well with each other, as the DI equals zero. In the nonlinear interaction case, the historical loads interact with each other along the temporal dimension, i.e., different loading history would give rise to diverse structural spectral responses. This aspect can be reflected in the frequency domain: DC responses exist in both frequency spectra; the spectral patterns become different between these two resonant responses among the target frequency ranges. As a consequence, the DI increased to around 0.01. Therefore, during the nonlinear ultrasonic resonance procedure, the spectral components would differ from each other under different loading directions.

4.4 Amplitude Dependence

The amplitude dependence of nonlinear response has been widely recognized and investigated for material nonlinearity problems. This case study further excavates the amplitude dependence feature for fatigue crack detection.

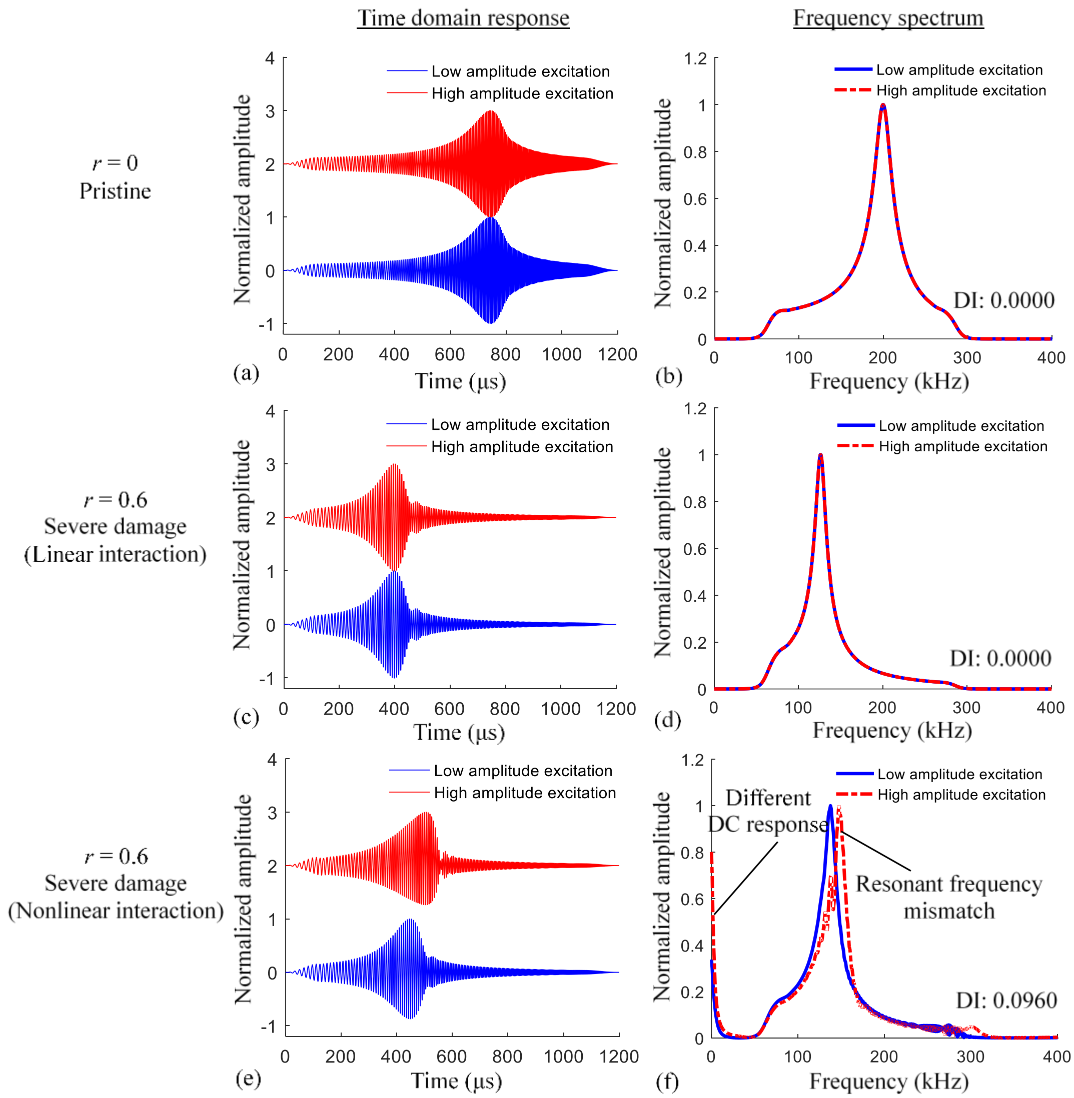

FIGURE 6: RESULTS OF AMPLITUDE DEPENDENCE CASE STUDY: (A) TEMPORAL SIGNAL OF THE PRISTINE CASE; (B) FREQUENCY SPECTRUM OF THE PRISTINE CASE; (C) TIME TRACE OF THE LINEAR WAVE DAMAGE INTERACTION CASE; (D) FREQUENCY SPECTRUM OF THE LINEAR WAVE DAMAGE INTERACTION CASE; (E) RESPONSE SIGNAL OF THE NONLINEAR WAVE DAMAGE INTERACTION CASE; (F) FREQUENCY SPECTRUM OF THE NONLINEAR WAVE DAMAGE INTERACTION CASE.

This case study keeps the sweeping direction the same but with different amplitudes. FIGURE 6 shows that the resonant responses under low and high amplitude excitations are the same in both time and frequency domain for both pristine and linear wave damage interaction cases; the only difference lies in that the resonant frequency is much lower when the damage is present. However, they become totally different in the nonlinear wave damage interaction case. While under larger amplitude excitation, more obvious CAN would be induced during the nonlinear resonance procedure. The mismatch of resonant frequency can be clearly identified from both the time trace signals and the frequency spectra. Furthermore, the DC response under high amplitude excitation is also much higher than that under low amplitude excitation. Such features could give rise to the DI value increasing up to about 0.1, which can help identify the nonlinear effects induced by the existence and severity of damage.

4.5 Breakage of Superposition

For linear vibration analysis, the superposition principle retains. However, when it comes to the nonlinear case, the superposition property would break down. This case study aims at exploiting the breakage of superposition feature for constructing another fatigue crack damage index.

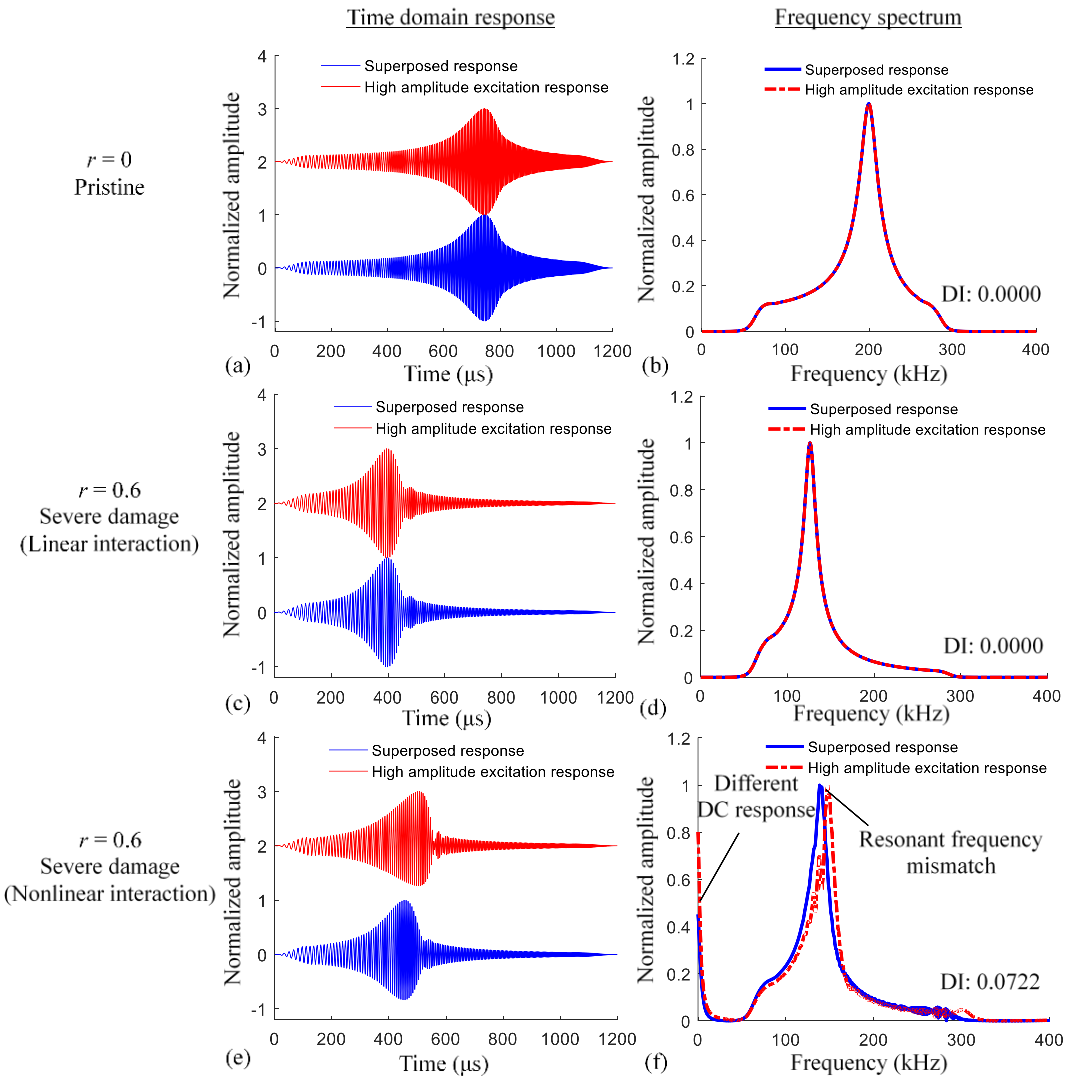

FIGURE 7: RESULTS OF BREAKAGE OF SUPERPOSITION CASE STUDY: (A) TEMPORAL SIGNAL OF THE PRISTINE CASE; (B) FREQUENCY SPECTRUM OF THE PRISTINE CASE; (C) TIME TRACE OF THE LINEAR WAVE DAMAGE INTERACTION CASE; (D) FREQUENCY SPECTRUM OF THE LINEAR WAVE DAMAGE INTERACTION CASE; (E) RESPONSE SIGNAL OF THE NONLINEAR WAVE DAMAGE INTERACTION CASE; (F) FREQUENCY SPECTRUM OF THE NONLINEAR WAVE DAMAGE INTERACTION CASE.

In this case study, three excitation signals are used, with the two comparatively low excitation amplitudes summing up to be the higher one. The temporal responses under low amplitude excitations were first superposed and then compared with the high amplitude response in the frequency domain. FIGURE 7 presents the results of the breakage of superposition case study. It can be clearly noticed that the superposition principle maintains in the pristine and linear wave damage interaction cases as both DIs equal to zero. Whereas in the nonlinear wave damage interaction case, the superposed response becomes different from the high amplitude excitation response, as different DC responses and resonant frequency mismatch can be obviously observed. Another remarkable feature lies in that the resonant frequency of high amplitude excitation response is higher than that of superposed response, which means that more nonlinear resonance features would take place under a larger amplitude excitation. Such phenomenon manifests the efficacy of the breakage of superposition feature for fatigue crack detection.

4.6 Multi-indicial Damage Quantification

These three case studies can work synthetically to evaluate the damage severities, which may form a more robust damage quantitative caliber. Here, the fused multi-indicial DI is further proposed for comprehensive fatigue crack detection and monitoring, which can be expressed as

$$DI_f = \sqrt{DI_1^2 + DI_2^2 + DI_3^2} \quad (6)$$

where DI_f denotes the fused damage index; DI_1, DI_2, and DI_3 represent the DI values, based on the time-history dependence, the amplitude dependence, and the breakage of superposition perspectives, respectively.

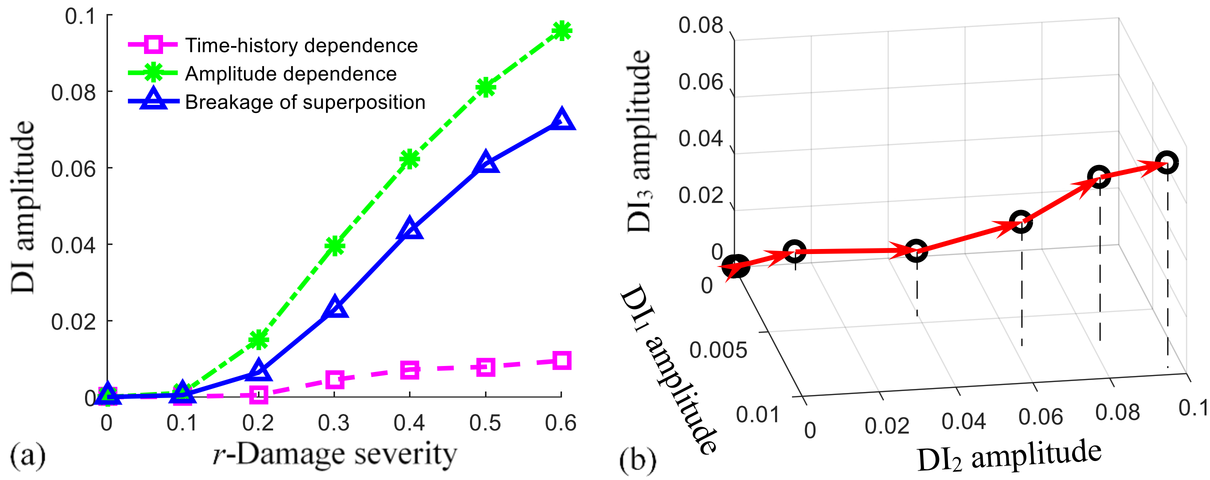

FIGURE 8: SYNTHETICALLY FUSED MULTI-INDICIAL DI: (A) DIS BASED ON THE THREE NONLINEAR RESONANCE FEATURES; (B) THE 3D PLOT OF THE FUSED DI.

FIGURE 8 illustrates the DI amplitudes from the perspectives of three nonlinear resonance features as well as the fused DI versus various damage severities. For the pristine condition, all the four DI values are zero. They increase a little in the small damage case (r =0.1), indicating the nucleation of fatigue crack. And all the three case study results present the same DI value in this case. Afterwards, they show a trend of rapid growth, where the nonlinear resonance features can be clearly distinguished. In details, the time-history dependence case study possesses the lowest DI value in both medium and severe damage conditions. The amplitude dependent DI presents the best performance for different crack sizes. The breakage of superposition case study results reveal the intermediate DI values in the medium and severe damage cases. From the 3D plot, it can be noticed that the fused DI starts from zero in the pristine case. Afterwards, it experiences an increment as the crack initiates, which demonstrates the superb sensitivity to the nucleation of fatigue cracks. With the increasing damage severity, the fused DI gradually drifts from the origin. The overall tendency indicates the spectral correlation based nonlinear ultrasonic resonance method could successfully monitor the growth of the fatigue crack. Once the damage severity rises beyond zero, the proposed technique could capture the nonlinear resonance features, exhibiting the increment of the fused DI value. And it is assumed that the corresponding severity represents the minimum flaw size that can be detected. Therefore, it can be concluded that both individual and fused DI can effectively detect and monitor the growth of fatigue cracks. For practical considerations, the fused DI will provide a more robust and reliable performance by taking on the sensitivity of each individual DI.

5. CONCLUDING REMARKS

This paper presented a spectral correlation based nonlinear ultrasonic resonance technique for fatigue crack detection. A reduced-order nonlinear oscillator model was tailored for illustrating the CAN and nonlinear resonance phenomenon considering the rough crack interface condition. The proposed technique further excavates various aspects of nonlinear resonance phenomena: (1) time-history dependence; (2) amplitude dependence; (3) breakage of superposition.

It was found that the improved reduced order model with the transitional rough crack region successfully captured the variable-stiffness nonlinear contact dynamics. The superharmonic nonlinear feature distinguished the resonance from the pristine case and an open notch case. It also showed the capability of nonlinear resonances for detecting the nucleation of fatigue cracks. Furthermore, it was demonstrated that the nonlinear feature became stronger with the increasing damage severity, presenting the great potential of nonlinear resonances for fatigue crack quantification. Three different perspectives of the nonlinear resonance phenomena can either work individually or collaborate synthetically to evaluate the existence and severity of fatigue cracks based on the spectral correlation algorithm.

The proposed nonlinear ultrasonic resonance technique are expected to be validated by the experimental demonstration in the future work. The specimen would be subjected to a cyclic tensile load, inducing the nucleation and propagation of fatigue cracks. During the fatigue test procedure, pitch-catch active sensing experiments should be simultaneously conducted to record the nonlinear resonances regarding to the three case studies. The system inherent nonlinearity may influence the nonlinear resonance responses, rendering the errors of DI value in each case study.

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