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# Application of Nonlinear-Modulation Technique for the Detection of Bolt Loosening in Frame Structure

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## Reference

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## ABSTRACT

Bolted joints are susceptible to loosening in the vibrational service environment, which would adversely affect the joint structure integrity. In this work, a nonlinear-modulation approach was explored as a potential method to effectively detect bolt loosening at its early stage, by analyzing the modulation spectrum that arises from the nonlinear vibrations caused by the loosening bolts. To reveal the mechanism of nonlinear modulation, a mathematical model was developed. An effective energy-based damage index was formulated based on the high-frequency intrinsic mode functions (IMF), which contains modulation components processed by empirical mode decomposition (EMD). Vibration tests on a frame structure were carried out to investigate the feasibility of the proposed method. The experimental results demonstrated that modulation occurred in the high-frequency region of response signals; it was found that the energy-based damage index can accurately detect nonlinear damage caused by bolt loosening with superb sensitivity.

## Keywords

bolt loosening, damage detection, vibration modulation, empirical mode decomposition, energy damage index, structural health monitoring

## Introduction

Bolted joints are widely used to connect structural components in mechanical and aerospace structures due to ease of assemblage and dismantlement. However, bolt loosening, induced by cyclic loading and environmental factors, would result in serious destruction of the integrity of structures. Thus, an effective approach for detecting bolt loosening is of great application significance in structural health monitoring (SHM) and non-destructive evaluation (NDE).

Nowadays, methods of loosening detection have been studied by many researchers. Among various damage detection methods, global methods based on the analysis of vibration data have received considerable attention. Hartwigsen et al. [1] reported an experimental study to quantify the non-linear effects of a typical shear lap joint. The tests revealed several important influences on the effective stiffness and damping of the lap joints. Milanese et al. [2] developed a joint loosening model to simulate the dynamic response to a stationary Gaussian excitation. Huda [3] proposed a vibration test methodology using an impulse response excited by laser ablation and introduced a damage index based on the Recognition Taguchi method to detect bolted joint loosening. Shiryayev et al. [4] developed an alternative parameter estimation approach for the adjusted Iwan, which was identified as a promising candidate for representing joint dynamics. Yu Luan et al. [5] proposed a nonlinear dynamic model with bi-linear springs validated for bolted pipe structures. Finally, experiments and numerical simulations are performed. Amerini et al. [6] presented a theoretical model of the interface stiffness where the stiffness was described as a function of the contact pressure. The theoretical model was verified well by the experimental data. Yang et al. [7] assessed the effectiveness of a method referred to as the adaptive quadratic sum-square error with unknown inputs for the damage assessment of joints in the frame structure. Amerini et al. [8] developed a damage index to access the connection state of a bolted structure using linear and nonlinear acoustic parameters. In the nonlinear methods, high-harmonics generation and sidebands modulation indices were established under multi-frequency excitation. Jaques et al. [9] explored impact modulation (IM) method to detect bolt loosening. Their results of IM testing and finite element models showed that IM is an effective method for differentiating the responses of the bolted structure under various bolt torque levels.

In general, when a connection becomes loose, nonlinearities may arise in the response signal. It is hardly to reap the nonlinear features extracted from the experimental signals [10]. Therefore, the time and frequency domain signal processing technique is considered more effective for detecting such a loosening status, with distinctive time-frequency nonlinear signatures. The empirical mode decomposition (EMD) technique was introduced by Huang et al. [11], as a new time-series analysis approach. It can be used to analyze the nonlinear and nonstationary data in the quest of accurate time and frequency localization. Since the EMD was proposed, it has received significant attention in structural health monitoring applications [12]. Loutridis [13] presented a method for monitoring the evolution of gear faults based on the empirical mode decomposition technique, establishing the modal energy for the correspondence of deterioration in the gear condition. Such a method is aimed toward system failure prediction, and this is done by establishing an energy-based damage index for the SHM purpose. Cheraghi et al. [14] put forward a novel damage index based on the first intrinsic mode functions (IMFs) after the EMD. Finite element analysis demonstrated that the evaluated damage index could effectively detect the defects, such as local corrosions. Through the analysis of vibration signals of a laboratory-scale single lap joint, Esmaeel et al. [15] found the energy damage index is feasible for the detection of bolt loosening. Razi et al. [16] demonstrated that the evaluated energy index based on the first and second IMFs could effectively distinguish various sizes of fatigue cracks.

It is clear from the above discussion that the EMD and energy damage index have been explored for damage detection in a variety of structural systems. However, the existing literatures only focused on applying the damage index to evaluate the structural health status. Few theoretical investigations have been reported to explain the rationale of the energy damage index established by the first or second IMF, caused by the loose bolts. When a loose bolt is present, additional sidebands can appear beside the excitation frequency, therefore forming a complex modulation spectrum. In this paper, a two df model is proposed to unfold the mechanism behind such nonlinear modulation phenomena via a multiple time scales method. In this model, a nonlinear stiffness is introduced to simulate the nonlinear feature caused by the bolts loosening. The presence of nonlinear components changes the characteristic of the response energy, which was used to develop a new damage index to evaluate the bolt loosening. Based on a bolted frame structure vibration tests, the power spectral density of the captured signals is calculated to analyze the phenomenon of nonlinear modulation in frequency domain. Ultimately, the effectiveness of the damage index established by the first IMF is verified for the detection of bolt loosening. Thus, the major contributions in this work can be identified as follows:

(1) Structural natural frequencies were utilized and incorporated into the nonlinear modulation phenomena
(2) A mathematical model was developed for explaining the modulation side band phenomena in nonlinear structural vibrations, revealing how the natural frequencies entered the final response spectrum as modulation components.
(3) Based on the energy density formulation, a new energy-based damage index was established for diagnosing bolt loosening status.
(4) The model and damage index were validated against the experimental results as an illustrative demonstration and educational example.

## Nonlinear Modulation Model and EMD Energy Damage Index

In this section, a mathematical model with quadratic stiffness is presented, adopting the multiple time scales method.

Theoretical fundamentals of the method are provided, followed by an explanation about the mechanism of nonlinear modulation and rationale of the proposed EMD energy damage index.

## NONLINEAR MODULATION THEORETICAL FORMULATION

Due to the presence of bolts loosening, the bolts' preload is reduced, resulting in the occurrence of relative movements between the joint parts, such as shear slide and axial impact. These localized relative movements between the joint parts can lead to the periodical changing of structural dynamic parameters (stiffness and damping) in vibrational cycles. As a consequence, the response contains complicated frequency components, involving distinctive nonlinear signatures, such as the frequency multiplication or modulation-induced sidebands of the excitation frequency. To model the dynamics of mechanical joints, the 2° spring mass system are always adopted in several studies, which performed effectively by experiments and numerical results [17]. In an effort to explore the mechanism of nonlinear modulation between excitation frequency and natural frequencies, analytical analysis is carried out using a 2 degree of freedom mathematical model. In this model, a nonlinear stiffness is introduced to represent the nonlinear feature of the system. It should be noted that this is a parametric model and that the stiffness is chosen by updating to the experiment results in the latter section. A quadratic form of contact stiffness was found to be best in matching with experimental data. The modulation sidebands appear at probing excitation frequency plus and minus the natural frequency, i.e., the natural frequency is the low frequency components that participate in the modulation.

**Fig. 1** shows a schematic of the mathematical model. The equation of motion for this model can be written as

$$\begin{cases} m_1\ddot{x}_1 + c_1\dot{x}_1 + k_2(x_1 - x_2) + k_1x_1 - k_1^*x_1^2 = Fe^{i(\Omega t+\phi)} \\ m_2\ddot{x}_2 + c_2\dot{x}_2 + k_2(x_2 - x_1) + k_3x_2 = 0 \end{cases} \quad (1)$$

where:

$m_1, m_2$ = the masses,

$k_1, k_2, k_3$ = the linear stiffness of the system, and

$c_1, c_2$ = damping.

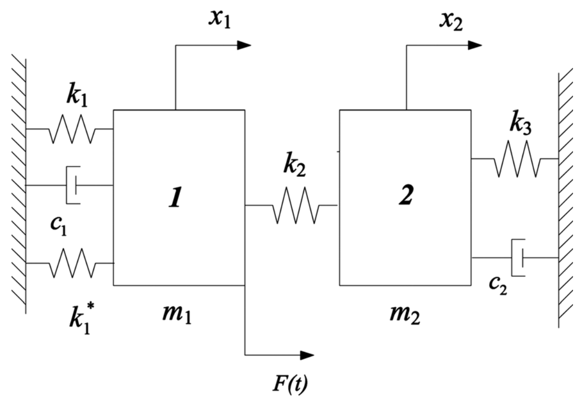
As mentioned previously, a quadratic form of nonlinearity is chosen, designated as $k_1^*$. Meanwhile, $F$ and $\Omega$ are the amplitude and frequency of a harmonic excitation. In order to simplify the derivation, it is assumed that $m_1 = m_2 = m$, $k_1 = k_2 = k_3 = k$, $k_1^* = \xi k$, $c_1 = c_2 = c$.

Equation 1 now becomes

$$\begin{cases} m\ddot{x}_1 + c\dot{x}_1 + k(2x_1 - x_2) - \xi k x_1^2 = Fe^{i(\Omega t+\phi)} \\ m\ddot{x}_2 + c\dot{x}_2 + k(2x_2 - x_1) = 0 \end{cases} \quad (2)$$

Firstly, decouple the motion differential equations of the model under the condition of linear free vibration

**FIG. 1** Schematic of the nonlinear 2 degree of freedom system.




$$\begin{cases} m\ddot{x}_1 + c\dot{x}_1 + k(2x_1 - x_2) = 0 \\ m\ddot{x}_2 + c\dot{x}_2 + k(2x_2 - x_1) = 0 \end{cases} \quad (3)$$

Two natural frequencies of the linear system are

$$\omega_1 = \sqrt{\frac{k}{m}}, \quad \omega_2 = \sqrt{\frac{3k}{m}} \quad (4)$$

Then, the normal mode shapes are

$$\phi = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (5)$$

Assume the following transformation on $x_1$ and $x_2$. The transformed displacements $u_1$ and $u_2$ are given

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \phi^{-1}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (6)$$

Substituting Eq 6 into Eq 2 yields:

$$\begin{cases} \ddot{u}_1 + \omega_1^2 u_1 = -2\mu\dot{u}_1 + \alpha u_1^2 + 2\alpha u_1 u_2 + \alpha u_2^2 + F_1 e^{i(\Omega t+\phi)} \\ \ddot{u}_2 + \omega_2^2 u_2 = -2\mu\dot{u}_2 + \alpha u_1^2 + 2\alpha u_1 u_2 + \alpha u_2^2 + F_1 e^{i(\Omega t+\phi)} \end{cases} \quad (7)$$

where: $\omega_1 = \sqrt{k/m}$, $\omega_2 = \sqrt{3k/m}$, $\mu = c/2m$, $\alpha = \xi k/\sqrt{2}m$, $F_1 = F/\sqrt{2}m$.

The method of multiple time scales as presented in Refs. [9,18] can be used to solve Eq 7. The first step in the multiple time scales method is to assume the following solution for $u$:

$$\begin{cases} u_1 = \varepsilon u_{11}(T_0, T_1) + \varepsilon^2 u_{12}(T_0, T_1) \\ u_2 = \varepsilon u_{21}(T_0, T_1) + \varepsilon^2 u_{22}(T_0, T_1) \end{cases} \quad (8)$$

where $T_0 = t$ and $T_1 = \varepsilon t$.

Assume $F_1 = \varepsilon f$ where the $\varepsilon$ indicates that a small force will result in a large response.

Substitute Eq 8 into Eq 7, noting that:

$$\frac{d}{dt} = \frac{\partial}{\partial T_0} + \varepsilon\frac{\partial}{\partial T_1} + \varepsilon^2\frac{\partial}{\partial T_2} + \cdots = D_0 + \varepsilon D_1 + \varepsilon^2 D_2 + \cdots \quad (9a)$$

$$\frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + \varepsilon^2\left(D_1^2 + 2D_0 D_2\right) + \ldots \tag{9b}$$

Then, the coefficients of $\varepsilon^0$ and $\varepsilon^1$ are equated to give:
$\varepsilon^0$:

$$\begin{cases} D_0^2 u_{11} + \omega_1^2 u_{11} = f e^{i(\Omega t + \phi)} \\ D_0^2 u_{21} + \omega_2^2 u_{21} = f e^{i(\Omega t + \phi)} \end{cases} \tag{10}$$

$\varepsilon^1$:

$$\begin{cases} D_0^2 u_{12} + \omega_1^2 u_{12} = -2D_0(D_1 u_{11} + \mu u_{11}) + \alpha u_{11}^2 + 2\alpha u_{11} u_{21} + \alpha u_{21}^2 \\ D_0^2 u_{22} + \omega_2^2 u_{22} = -2D_0(D_1 u_{21} + \mu u_{21}) + \alpha u_{11}^2 + 2\alpha u_{11} u_{21} + \alpha u_{21}^2 \end{cases} \tag{11}$$

The general solution of Eq 10 is

$$\begin{cases} u_{11} = A_1(T_1) e^{i\omega_1 T_0} + \Lambda_1 e^{i\Omega T_0} + cc \\ u_{21} = A_2(T_1) e^{i\omega_2 T_0} + \Lambda_2 e^{i\Omega T_0} + cc \end{cases} \tag{12}$$

where:

$cc$ = the complex conjugate of the previous terms,
$A_1, A_2$ = the complex amplitude of free vibration, and
$\Lambda_1, \Lambda_2$ matches the forced response of a linear system.
Therefore,

$$\Lambda_1 = \frac{f}{2(\omega_1^2 - \Omega^2)}, \quad \Lambda_2 = \frac{f}{2(\omega_2^2 - \Omega^2)} \tag{13}$$

Next, substitute Eq 12 into Eq 11:

$$\begin{aligned} D_0^2 u_{12} + \omega_1^2 u_{12} &= -2D_0(D_1 + \mu) A_1 e^{i\omega_1 T_0} - 2D_0(D_1 + \mu)\Lambda_1 e^{i\Omega T_0} \\ &+ \alpha A_1^2 e^{2i\omega_1 T_0} + \alpha A_2^2 e^{2i\omega_2 T_0} + \alpha(\Lambda_1 + \Lambda_2)^2 e^{2i\Omega T_0} \\ &+ 2\alpha(\Lambda_1 + \Lambda_2) A_1 e^{i(\omega_1 + \Omega)T_0} + 2\alpha(\Lambda_1 + \Lambda_2) A_2 e^{i(\omega_2 + \Omega)T_0} \\ &+ 2\alpha(\Lambda_1 + \Lambda_2)\bar{A}_1 e^{i(\Omega - \omega_1)T_0} + 2\alpha(\Lambda_1 + \Lambda_2)\bar{A}_2 e^{i(\Omega - \omega_2)T_0} \\ &+ 2\alpha A_1 A_2 e^{i(\omega_1 + \omega_2)T_0} + 2\alpha \bar{A}_1 A_2 e^{i(\omega_2 - \omega_1)T_0} + cc \end{aligned} \tag{14a}$$

$$\begin{aligned} D_0^2 u_{22} + \omega_1^2 u_{22} &= -2D_0(D_1 + \mu) A_2 e^{i\omega_2 T_0} - 2D_0(D_1 + \mu)\Lambda_2 e^{i\Omega T_0} \\ &+ \alpha A_1^2 e^{2i\omega_1 T_0} + \alpha A_2^2 e^{2i\omega_2 T_0} + \alpha(\Lambda_1 + \Lambda_2)^2 e^{2i\Omega T_0} \\ &+ 2\alpha(\Lambda_1 + \Lambda_2) A_1 e^{i(\omega_1 + \Omega)T_0} + 2\alpha(\Lambda_1 + \Lambda_2) A_2 e^{i(\omega_2 + \Omega)T_0} \\ &+ 2\alpha(\Lambda_1 + \Lambda_2)\bar{A}_1 e^{i(\Omega - \omega_1)T_0} + 2\alpha(\Lambda_1 + \Lambda_2)\bar{A}_2 e^{i(\Omega - \omega_2)T_0} \\ &+ 2\alpha A_1 A_2 e^{i(\omega_1 + \omega_2)T_0} + 2\alpha \bar{A}_1 A_2 e^{i(\omega_2 - \omega_1)T_0} + cc \end{aligned} \tag{14b}$$

Next, eliminate the terms which would lead to secular terms in the solution. Therefore,

$$-2D_0(D_1 + \mu) A_1 e^{i\omega_1 T_0} = 0 \tag{15a}$$

$$-2D_0(D_1 + \mu) A_2 e^{i\omega_2 T_0} = 0 \tag{15b}$$

To solve Eq 15a for $A_1$, first assume that

$$A_1 = \alpha e^{i\beta}, \text{ where } \alpha = \alpha(T_1) \text{ and } \beta = \beta(T_1) \tag{16}$$

Substituting Eq 16 into Eq 15a, the steady state amplitude, $A_1(T_1)$, can be obtained. Then, the approximate response solution of Eq 14a can be determined as:

$$\begin{aligned} u_{12} &= \psi_1 e^{i\Omega T_0} + \psi_2 e^{2i\omega_1 T_0} + \psi_3 e^{2i\omega_2 T_0} + \psi_4 e^{2i\Omega T_0} \\ &+ \psi_5 e^{i(\omega_1 + \Omega)T_0} + \psi_6 e^{i(\omega_2 + \Omega)T_0} + \psi_7 e^{i(\Omega - \omega_1)T_0} \\ &+ \psi_8 e^{i(\Omega - \omega_2)T_0} + \psi_9 e^{i(\omega_1 + \omega_2)T_0} + \psi_{10} e^{i(\omega_2 - \omega_1)T_0} \end{aligned} \tag{17}$$

where $\psi_i$ and $\varphi_i$ ($i = 1,2,\ldots,10$) represent the amplitudes of frequency components, respectively. Meanwhile, the solution of $u_{22}$ can be obtained in same way.

The mathematical model presented above is a simplified reduced order model to help us to understand the principle phenomena and physics behind the proposed nonlinear modulation inspection technique. The derivation for the response of a system with a quadratic nonlinearity shows that the second order solutions in Eq 17 contain frequency components ($\Omega \pm \omega_1$, $\Omega \pm \omega_2$), equal to the excitation frequency ($\Omega$) plus and minus the natural frequencies ($\omega_1, \omega_2$). This explains the appearance of additional sidebands theoretically and demonstrates that the energy distribution of the response is changed. Thus, the method by comparing the energy damage index is of great potential to evaluate the status of bolted joints. Meanwhile, the dependence of the sideband frequency with excitation frequencies and natural frequencies suggests that more complexity may arise due to this dependence in a multi-degree of freedom system in which it is possible to have more various combinations in the frequency range of the sidebands. To address this additional complexity and to explore the effects of the parameters of a nonlinear system, a vibration experiment of a frame structure will be presented in the later section.

In general, high frequency parts such as those near the excitation frequency and within the modulation range are sensitive to the appearance of nonlinear damage. The modal vibrations at low frequencies, by themselves, are not sensitive to changes of the clamping force at the bolted joint. Admittedly, the sensitivity will increase with the increment of load intensity at the low frequency range. However, in this study, the focus is on a nonlinear modulation technique, where low frequency pumping vibrations and high frequency probing excitation work together, i.e., the excitation signal is selected as the combination of a high frequency sine and a band limited random input. Since the modulation sidebands always appear near the high frequency probing excitation, the effect of structural nonlinearity will be most prominent around the relatively high frequency range. Thus, in this work, the original signal is decomposed by EMD firstly. Afterwards, an efficient damage index is established using the high-frequency parts of the data that contains the nonlinear modulation components

### AN ENERGY BASED DAMAGE INDEX

By the process of EMD, IMFs are extracted from the original signal. Through the local maxima and minima of the time signal, two cubic splines are fitted to produce an upper and lower envelope, respectively. Then, the average of the two gained is subtracted from the original signal. Based on the EMD approach described above, the first IMF contains the shortest period component of the signal. The resultant signal is subsequently treated as a new one, on which the aforementioned shifting process is applied to obtain the second IMFs. This procedure is repeated for all subsequent residues to derive the lower frequency components. Finally, the original signal can be represented by the following mathematical relation:

$$X(t) = \sum_{i=1}^{n} C_i(t) + r_n \tag{18}$$

where $C_i$ represents the $i$th IMF and $r_n$ means a residue.

Based on this decomposition method, a damage index based on the energy of IMFs was used to analyze the vibration-based signals for structural health monitoring. The damage index, referred to as EMD_E, is based on the energy of certain IMFs of a given vibration signal. After the structure is excited, the dynamic response (e.g., acceleration signal) is obtained through appropriate sensors. After the IMFs are established, the energy of the desired IMF is defined as (hereafter referred to as EMD energy)

$$E = \int_0^{t_0} (\mathrm{IMF})^2 \mathrm{d}t \tag{19}$$

The damage index is defined as

$$\mathrm{EMD\_E} = \left| \frac{E_h - E_d}{E_h} \right| \times 100 \tag{20}$$

where $E_h$ and $E_d$ represent the energy of the desired IMFs from the intact (healthy) structure and the damaged (bolt loosening) structure, respectively.

The first IMF contains the oscillatory modes of higher frequencies of a signal. Generally, high frequency components are believed to be more sensitive to the appearance of small-scale localized nonlinear damage. Thus, all of the output signals of sensors would pass through the procedure of EMD, and the EMD_E damage index is based on the first IMF of high frequency components.

## Experimental Study

Experimental case studies are carried out to examine the capability of the proposed method. The power spectral density (PSD) derived from experimental data are presented for various joint conditions to assess the presence of additional sidebands. Ultimately, EMD_E damage index is established to evaluate the bolt loosening status.

### EXPERIMENTAL SETUP

The vibration test setup is presented in Fig. 2a. The bolted structure that consists of steel columns and floors is excited laterally via an electrodynamic shaker (MB model50) at the first floor.

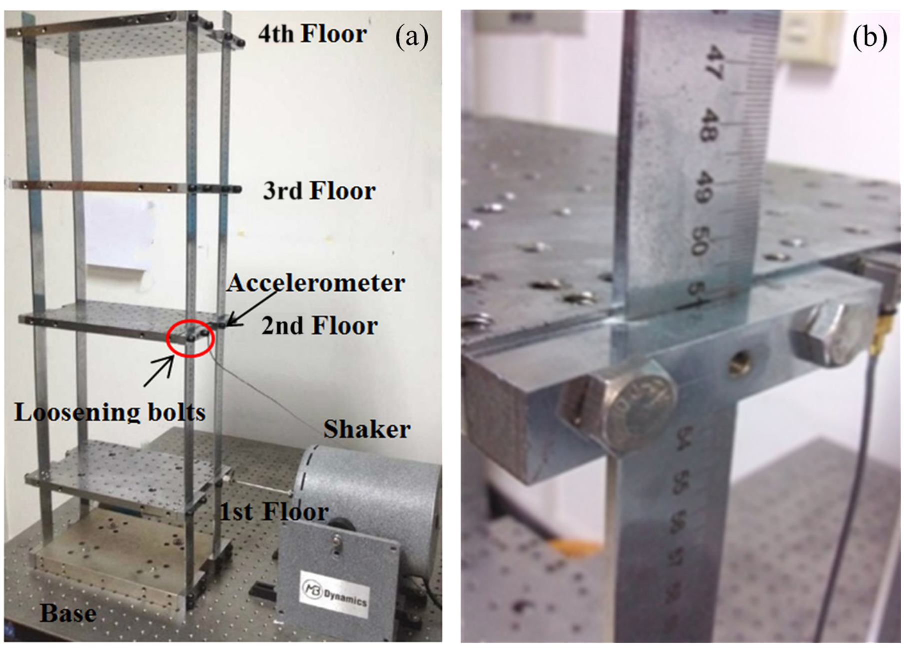



**FIG. 2**

Experimental setup: (a) the frame structure and excitation device; (b) position of the lessening bolts.

The structure and the shaker are mounted together on a baseplate. The damage is introduced as a progressive loosening of the bolt in the second floor, shown in **Fig. 2b**.

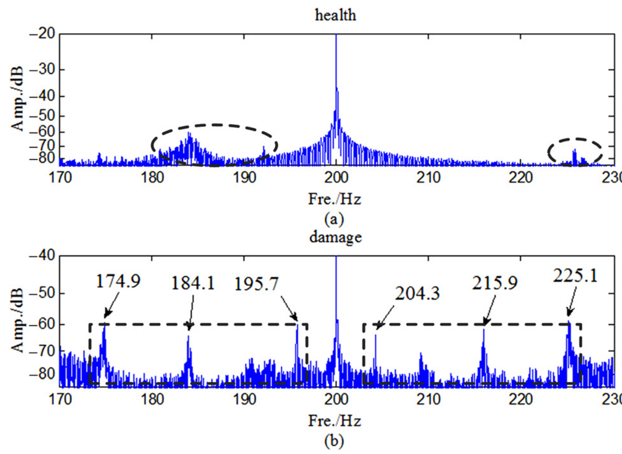
The excitation signal is the combination of a high frequency input and a band limited random excitation, generated by an arbitrary-function generator (Agilent33522A). Considering the structure's low frequency modes, the random excitation is chosen in the range of 1–150 Hz. Meanwhile, the high frequency input of the excitation is chosen at 200 Hz. The low-frequency band limited random excitation is used to excite the structure's low frequency modes as the pumping signal, while the high-frequency sine waves are used as the probing signal, which is distorted and mixed with the low frequency components at a nonlinear damage. The proper choice of the pumping and probing frequency is of importance to generate the nonlinear modulation. In this study, the natural frequencies of the first three modes of tested structure are under 30 Hz, which are used as the pumping signal. Thus, a probing signal at 200 Hz, far away from the first three natural frequencies, renders sufficient distance in the spectrum to display easily-identifiable modulation components. The mixed frequency response (nonlinear modulations) will only happen in nonlinear systems, while it will not present in a linear system. To generate the phenomena of nonlinear modulation, other forms of excitation signal could also be used as effective method to excite the low frequency modes, such as linear chirp excitation. Nevertheless, the main purpose of these excitation profiles is to excite the vibration modes at the natural frequencies as the pumping signal. An accelerometer (B&K 4507B) is attached at the centerline of the second floor to collect the dynamic response of the system. The response signals are collected and processed by an NI PXIe-1071 data acquisition system. Time test duration is 40 s in this work.

Four joint conditions are considered, which can be categorized into two main groups. The first group is regarded as a baseline condition with tight bolts, as the reference structural state (case 1). The acquired response signal is regarded as the healthy status to calculate $E_h$, which is EMD energy of the intact structure. The second group is the states with loose bolts. The bolts are adjusted to different torque levels by torque wrench successively in order to simulate the damaged states, which is 23 N m(case 2), 19 N m(case 3), and 0 N m(case 4). In the damage cases, the EMD energies of damaged structural status are calculated as $E_d$.

**TABLE 1** Natural frequencies of the structure/Hz.

| Mode | Case 1 | Case 2 | Case 3 | Case 4 |
|---|---|---|---|---|
| 1 | 4.50 | 4.27 | 4.29 | 4.36 |
| 2 | 16.10 | 15.99 | 15.95 | 15.96 |
| 3 | 25.65 | 25.19 | 25.15 | 25.40 |

**FIG. 3** High-frequency part of PSD from intact and damage response signals: (a) the intact base-line situation with the inherent nonlinear components highlighted by a dotted ellipse (case 1); (b) the bolt loosening damaged situation with the sidebands highlighted by the dotted box (case 3).




## ANALYSIS OF NONLINEAR MODULATION SIGNALS

Firstly, the power spectral density of the response is calculated. **Table 1** reports the first three natural frequencies in each case. For example, the natural frequencies in case 3 is 4.29, 15.95, and 25.15 Hz, designated as $f_1$, $f_2$, and $f_3$. The changes in the natural frequencies showed little sensitivity to the loosening damage, while the EMD_E index shown in the next section will demonstrate superb sensitivity for damage detection.

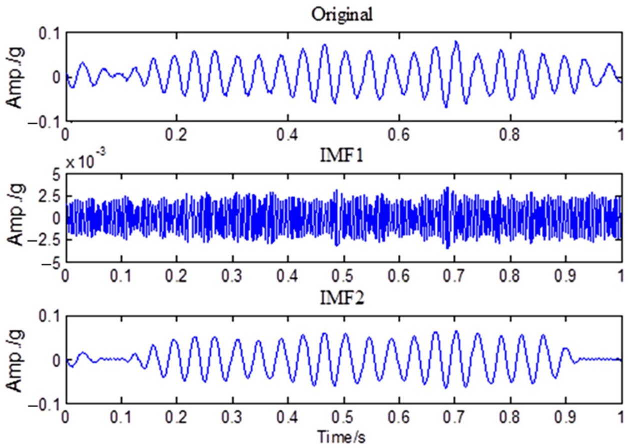
**Fig. 3** shows the high-frequency part of the spectra obtained by applying PSD to the signal obtained from case 1 and case 3. As is presented in **Fig. 3a**, the high-frequency spectrum in the intact case contains a few nonlinear components beside the excitation frequency, and it is influenced by test environment and structural inherent nonlinearity. **Fig. 3b** shows that, for the damaged case with bolt loosening, distinctive additional sidebands occurred in the high frequency spectrum near the excitation frequency. These frequencies correspond to 200 Hz, the excitation frequency, plus and minus each natural frequency. For example, the frequency component at 215.9 Hz is the sum of excitation frequency (200 Hz) and the third natural

**TABLE 2** Combinations of the harmonic excitation frequency ($f_0$) and natural frequencies ($f_1$, $f_2$, $f_3$).

$f_0 = 200$ Hz, $f_1 = 4.29$ Hz, $f_2 = 15.95$ Hz, $f_3 = 25.15$ Hz

| Frequency/Hz | Combination |
|---|---|
| 174.9 | $f_0 - f_3$ |
| 184.1 | $f_0 - f_2$ |
| 195.7 | $f_0 - f_1$ |
| 204.3 | $f_0 + f_1$ |
| 215.9 | $f_0 + f_2$ |
| 225.1 | $f_0 + f_3$ |

**FIG. 4** Original signal and first two IMFs for case 2: (a) response of original signal; (b) response of the first IMF; (c) response of the second IMF.




frequency (15.95 Hz). **Table 2** presents the six combinations of modulation. This clearly shows that modulation in the response could be observed in the damaged experimental case related to the nonlinearity due to bolt loosening, while such phenomenon does not happen in the tight bolt case. The result of this part of the study confirms the fact that the frequency modulation analysis could be a sensitive means for detecting bolts loosening.

The EMD algorithm was coded using MATLAB to process the experimental data. **Fig. 4** shows original response of case 2 and its first two IMFs. It is obvious from the figure that the high frequency part is extracted from the original response as the first IMF, while the second IMF contains the low frequency part. The spectrum of original signal and first two IMFs are showed in **Fig. 5**. As is shown in **Fig. 5a**, the original signal contains three natural frequencies in the range from 0 to 40 Hz, which decrease greatly in the first IMF. However, the high frequency spectrum derived by the first IMF is quite similar to that of the original signal, which retains the sidebands completely. In comparison, the excitation frequency (200 Hz) and nonlinear frequencies in sidebands are eliminated in the second IMF. It indicates that the first IMF can be used to reflect the substantial characteristics of nonlinear damage.

## EMD_E DAMAGE INDEX

Based on the method of the EMD_E, the energy damage index established by the first IMF are 4.35, 204.58, 196.62, and 20.80 for these four cases, shown in **Fig. 6**. The results reveal that the index can successfully detect the presence of bolt loosening. For instance, the indexes in the damage cases exceed 20, which means the index in the intact state is only 4.35. By comparing changes in the natural frequencies, the EMD_E index is more effective to show the presence of damage. As shown in the "Analysis of Nonlinear Modulation Signals" section, the response of an intact structure contains less modulation than the response of a structure with bolt loosening. When a connection comes loose, the interaction of joint interfaces results in the appearance of nonlinearity in the sutural response. These changes of response make the EMD energy in the damaged case differ from the healthy case, which result in the increase of EMD energy index finally.

**Fig. 6** shows that the indices of case 2 and case 3 are not in coordination with the index of case 4, which is much lower. The torque levels of the case 2 and case 3 are 23 N m and 19 N m, regarded as the early loosening state. In the early loosening state, the reduced preload and short distance of

**FIG. 5** PSD of original signal and first two IMFs: (a) low-frequency part of the original signal; (b) low -frequency part of the first IMF; (c) low -frequency part of the second IMF; (d) high -frequency part of the original signal with the sidebands and probing highlighted by a dotted box and a dotted ellipse;(e) high -frequency part of the first IMF highlighted by a dotted box and a dotted ellipse; (f) high -frequency part of the second IMF.

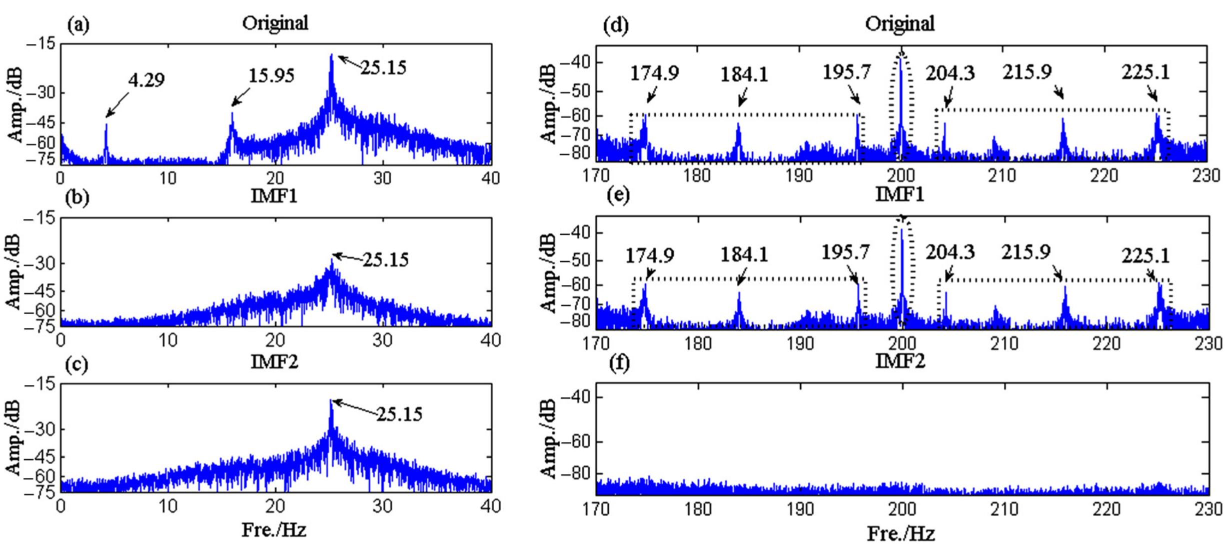
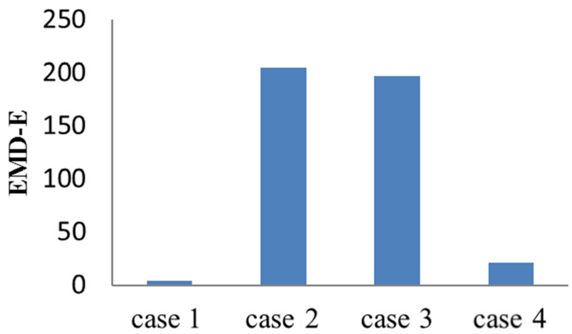


**FIG. 6** EMD_E evaluated based on the first IMF.




joint interface will result in slip and collision of the interface between joint elements. These nonlinear behaviors make the structural stiffness and damping changed periodically during the vibration. And this is the reason why the nonlinear modulation occurred in the response signals and the EMD_E damage indices increased greatly. However, when the bolt is completely removed, such as the case 4, the local nonlinearity (relative movements between the joint parts) of the structure become weak. Therefore, the whole dynamic system approaches a linear one at this end. The damage index of the case 4 becomes much lower due to the decrease of nonlinear behaviors. It was found that the method based on EMD_E damage index is sensitive to the loosening damage that occurred in the early stage, which is the focus and aim of monitoring/detecting such incipient changes before catastrophic failure happens. From the experimental results and the analysis of nonlinear vibration modulation model, the proposed EMD_E method proved to be an efficient way to detect the nonlinear damage caused by bolt loosening.

## Concluding Remarks

This study examined and investigated the application of a nonlinear vibration method based on EMD and modulation phenomenon for the effective detection of bolt loosening at its early stage. Through the analyses of a 2 degree-of-freedom mathematical model, additional sidebands in the response were found to appear near the excitation frequency, induced by a quadratic nonlinear stiffness. The results indicate that the nonlinearities associated with system parameters will cause the nonlinear modulation in structural vibrations. A vibration test on frame structure was carried out to show the applicability of the proposed method. The experimental result demonstrated that the modulation sideband appeared in the response caused by loose bolts as predicted by the theoretical formulations, and the new energy-based damage index developed in this study can be used to accurately detect nonlinear damage. Moreover, the EMD_E damage index is more sensitive to the early bolts loosening state, which will ultimately prevent the happening of uncontrollable catastrophic failures.

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