

COVER SHEET

Title: Predictive modeling of ultrasonics SHM with PWAS transducers

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ABSTRACT¹

This paper presents a set of analytical and numerical results on the use of guided waves for structural health monitoring application. The aim of the work presented in this paper is to provide tools to extend modeling capacities and improve quality and reliability of guided wave propagation models using commercially available finite element (FE) packages. Predictive simulation of ultrasonic non-destructive evaluation (NDE) and structural health monitoring (SHM) in realistic structures is challenging. The principle of guided wave propagation is studied and an analytical model is developed to obtain the analytical waveform and theoretical frequency contents solution of arbitrary engineering situations. Two benchmark problems, one 1-D and the other 2-D to achieve reliable and trustworthy predictive simulation of ultrasonic guided wave SHM with FEM codes have been proposed. The proposed 1-D benchmark problem will be a simple pitch-catch arrangement between a transmitter PWAS and receiver PWAS in a plate. The proposed 2-D benchmark problem will be the pitch-catch arrangement in a full 2-D plate involving circular-crested guided waves between a transmitter PWAS and receiver PWAS. In addition, a circular through hole of defined size ratio could be added to the 2-D problem to simulate the detection of damage, and assess the detectability threshold since closed-form solution exists for this problem too. The paper finishes with summary, conclusion, and suggestions for further work.

INTRODUCTION

Predictive simulation of ultrasonic non-destructive evaluation (NDE) and structural health monitoring (SHM) in realistic structures is challenging [1]. Analytical methods (e.g., ray-tracing, beam/pencil, Green functions, etc.) [2] can perform efficiently modeling of wave propagation but are limited to simple geometries.

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Realistic structures with complicated geometries are usually modeled with the finite element method (FEM). Commercial FEM codes offer convenient built-in resources for automated meshing, frequency analysis, as well as time integration of dynamic events. Even a relatively rough FE model would yield a ‘wave propagation’ animated output that is illustrative and interesting to watch. However, to obtain accurate wave propagation solution at ultrasonic frequencies is computationally intensive and may become prohibitive for realistic structures. Several investigators have previously addressed the convergence of FEM solutions for NDE-type ultrasonic inspection (bulk waves) and have developed useful guidelines [3, 4]. This paper addresses this issue in the context of guided-waves for SHM with piezoelectric wafer active sensors (PWAS) using the pitch-catch method.

ANALYTICAL MODELING

The analytical modeling is carried out in frequency domain, and could be described into four steps:

- 1) The excitation signal $V_e(t)$ is Fourier transformed into $V_e(\omega)$;
- 2) The plate transfer function in frequency domain is obtained as $G(\omega)$;
- 3) The excitation signal and the plate transfer function are multiplied to obtain the receiver sensing signal in frequency domain $V_r(\omega) = V_e(\omega) \cdot G(\omega)$;
- 4) The receiver sensing signal is inverse Fourier transformed back into time domain and the waveform in time domain is obtained by:

$$V_r(t) = \text{IFFT}\{V_r(\omega)\} = \text{IFFT}\{V_e(\omega) \cdot G(\omega)\} \quad (1)$$

where IFFT denotes inverse Fourier transform, $G(\omega)$ is the frequency-dependent structure transfer function that affects the wave propagation through the medium. In this paper, the main interest is in symmetric fundamental mode (S_0) and anti-symmetric fundamental mode (A_0). For Lamb waves with only two modes (A_0 and S_0) excited, the structure transfer function $G(\omega)$ can be derived from:

$$\begin{aligned} \varepsilon_x(x, t) \Big|_{y=d} = & -i \frac{a\tau_0}{\mu} (\sin k^S a) \frac{N_S(k^S)}{D_S(k^S)} e^{-i(k^S x - \omega t)} \\ & - i \frac{a\tau_0}{\mu} (\sin k^A a) \frac{N_A(k^A)}{D_A(k^A)} e^{-i(k^A x - \omega t)} \end{aligned} \quad (2)$$

So, $G(\omega)$ can be written as

$$G(\omega) = S(\omega) e^{-ik^S x} + A(\omega) e^{-ik^A x} \quad (3)$$

With

$$S(\omega) = -i \frac{\alpha \tau_0}{\mu} (\sin k^S a) \frac{N_S(k^S)}{D_S'(k^S)} \quad (4)$$

$$A(\omega) = -i \frac{\alpha \tau_0}{\mu} (\sin k^A a) \frac{N_A(k^A)}{D_A'(k^A)} \quad (5)$$

$$D_S = (k^2 - \beta^2)^2 \cos(\alpha d) \sin(\beta d) + 4k^2 \alpha \beta \sin(\alpha d) \cos(\beta d) \quad (6)$$

$$D_A = (k^2 - \beta^2)^2 \sin(\alpha d) \cos(\beta d) + 4k^2 \alpha \beta \cos(\alpha d) \sin(\beta d) \quad (7)$$

$$N_S = k \beta (k^2 - \beta^2) \cos(\alpha d) \sin(\beta d) \quad (8)$$

$$N_A = k \beta (k^2 - \beta^2) \sin(\alpha d) \cos(\beta d) \quad (9)$$

$$\alpha^2 = \frac{\omega^2}{c_p^2} - k^2 \quad (10)$$

$$\beta^2 = \frac{\omega^2}{c_s^2} - k^2 \quad (11)$$

where a is the half length of the PWAS, τ_0 is the shear stress between PWAS and the plate, μ is Lamé's constant, k^S and k^A are the wavenumbers for S_0 and A_0 respectively, x denotes the distance between the two PWAS transducers, k represents the wavenumber for S_0 or A_0 accordingly, and c_p and c_s are the wave speed for pressure wave and shear wave respectively. In the transfer function, it could be observed that $S(\omega)$ and $A(\omega)$ will determine the amplitude of S_0 and A_0 mode. In both $S(\omega)$ and $A(\omega)$ terms, there is $\sin(k^S a)$ and $\sin(k^A a)$, in which it is lies to the tuning effect.

The wave speed dispersion curve is obtained by solving Rayleigh-Lamb equations, which are transcendental equations that require numerical solution. The usual form of Rayleigh Lamb equations are as follows:

$$\frac{\tan(\beta d)}{\tan(\alpha d)} = -\frac{4\alpha\beta k^2}{(k^2 - \beta^2)^2} \quad (12)$$

$$\frac{\tan(\beta d)}{\tan(\alpha d)} = -\frac{(k^2 - \beta^2)^2}{4\alpha\beta k^2} \quad (13)$$

where the plate thickness is $2d$. After getting the wave speed dispersion curve, the wavenumber for each frequency component i.e $\xi = \frac{\omega}{c}$ is known. Thus, all the terms involved in the plate transfer function could be solved, and the plate transfer function $G(\omega)$ is obtained. After the plate transfer function $G(\omega)$ is obtained, the excitation signal is Fourier transformed.

1D FINITE ELEMENT MODELING

In the context of wave propagation modeling, the choice of the solving technique, mesh density and time step influences the successful outcomes of the exercise but also the level of accuracy with which the phenomenon is represented. For time domain models solved with an explicit solver, we investigate the influence of the mesh density for linear quadrilateral elements for both A_0 and S_0 modes waves using the commercial software Abaqus. Both S_0 and A_0 wave generations are considered, as models excited by two nodal forces. The distance between the both forces nodes correspond to the real values of a PWAS. The time domain excitation signal considered in our studies consisted of a 150 kHz three tone burst filtered through a Hanning window. The distance between the transducer and the receiver is 100 mm. The mesh density is expressed as $N = \lambda/L$ in terms of nodes per wavelength, with λ is the wavelength and L is the size of the square element. The Figure 1 highlights the strong influence of the mesh density on the group velocity error for the A_0 mode. The curve shows how the error varies from a value of about 9% for $N=15$ to a value of 0.2% for $N=254$ for this mode. For the fundamental symmetric mode S_0 , the error varies from a value of about 2% for $N=20$ to a value of 0.15% for $N=120$.

As mentioned previously, the mesh density value has a great impact on the computational model size and therefore the amount of memory required solving the model.

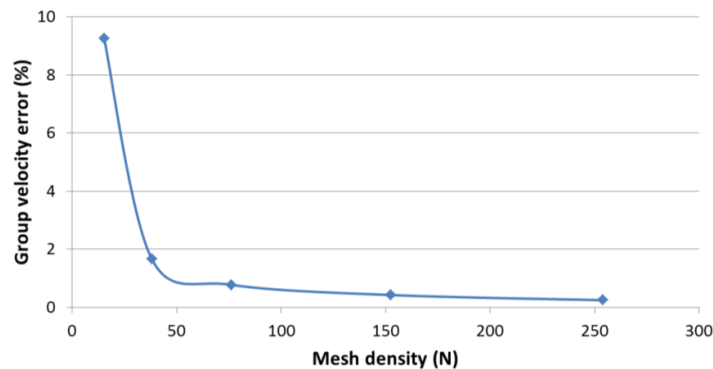


Figure 1: Group velocity error versus the mesh density for the A_0 mode.

RESULTS AND DISCUSSION

The analytical modeling, the finite element modeling and the experimental results of a 1-mm thick aluminum plate with 100-mm PWAS distance for a frequency of 150 kHz are shown in Figure 2. S_0 and A_0 mode wave packages could be observed. The wave speed of S_0 mode is higher than the A_0 mode, so the S_0 wave packet is picked up earlier than the A_0 wave packet.

The excitation signal having a center frequency of 150 kHz, we would have thought that the center frequencies of the two fundamental modes are also 150 kHz. In fact, as shown in Figure 3, this is not the case. We note that the center frequency of the A_0 mode undergone a shift towards lower frequencies while the S_0 mode undergone a shift towards higher frequencies.

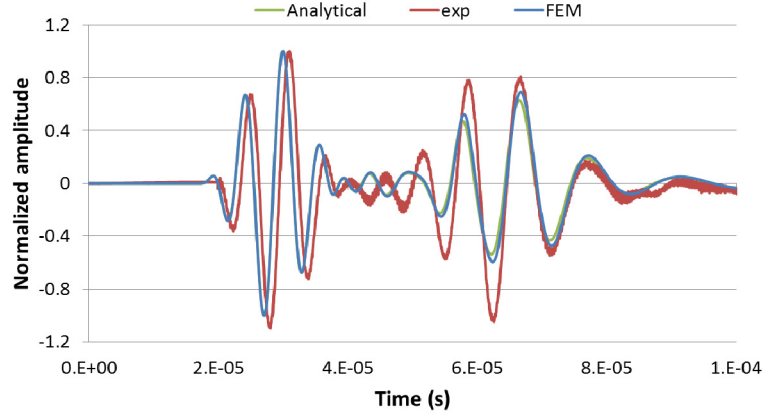


Figure 2: Comparison between the analytical, the FEM and the experimental receive signal from the PWAS for a 150 kHz frequency

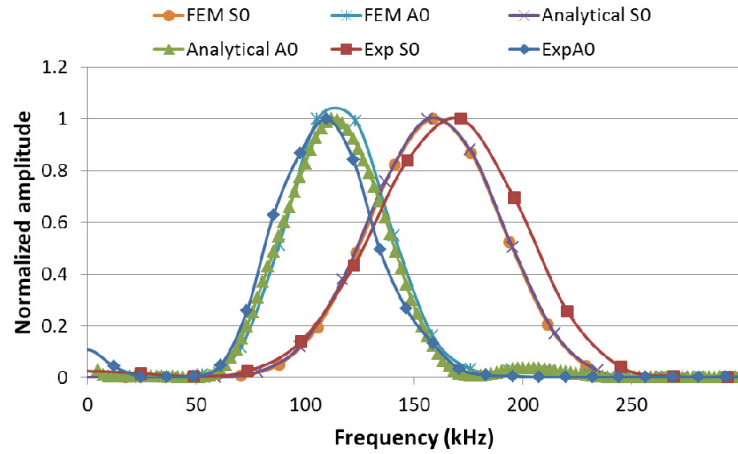


Figure 3: FFT of the S0 wave packet and the A0 mode packet for the analytical, FEM and experimental signal received.

A theoretical solution for the magnitude of frequency contents for S0 and A0 could be derived from equation 4 and 5 after discarding the same factor which does not influence amplitude relation:

$$|V_{sr}(\omega)| = \left| \left(\sin k^S a \right) \frac{N_S(k^S)}{D'_S(k^S)} \right| |V_e(\omega)| \quad (15)$$

$$|V_{Ar}(\omega)| = \left| \left(\sin k^A a \right) \frac{N_A(k^A)}{D'_A(k^A)} \right| |V_e(\omega)| \quad (16)$$

So, the amplification coefficients for each package are:

$$C_S = \left| \left(\sin k^S a \right) \frac{N_S(k^S)}{D'_S(k^S)} \right| \quad (17)$$

$$C_A = \left| \left(\sin k^A a \right) \frac{N_A(k^A)}{D'_A(k^A)} \right| \quad (18)$$

The amplification coefficients are directly related to the PWAS size a , material properties, plate thickness and the corresponding frequency. A theoretical solution is obtained for a 150 kHz Hanning window modulated sine tone burst excitation signal by a 7-mm PWAS transducer coupling with a 1-mm thick Aluminum plate. The frequency contents of A0 and S0 pitch-catch signals are shown in Figure 4.

When modeling by finite element and analytical model simulates the PWAS transducer by a single point force, i.e., $a \rightarrow 0$, the frequency shift becomes zero.

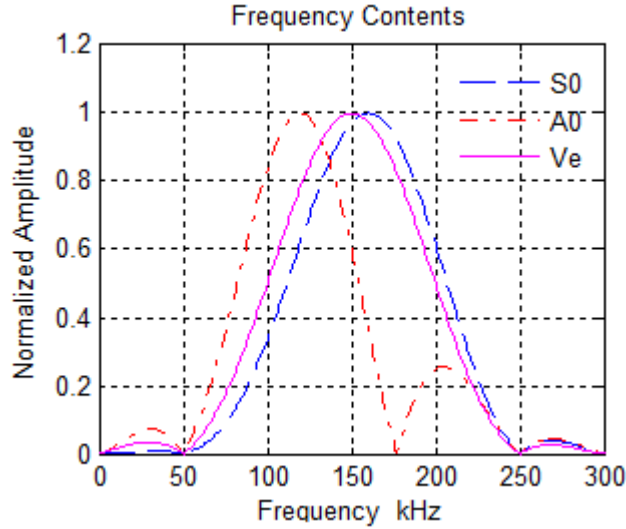


Figure 4: Frequency contents of S0 and A0 packets and excitation Ve at 150 kHz.

2D FINITE ELEMENT MODELING

The use of multi-physics finite element method (MP-FEM) to model the generation of elastic waves from an applied electric field applied to a surface-mounted PWAS transmitter (T-PWAS) and the reception of the elastic wave as electric signal recorded at a PWAS receiver (R-PWAS) have been explored. A study on modeling the guided wave generation and reception in a rectangular metallic plate containing a through-hole defect has been performed. The 7-mm PWAS transducers bonded to the top of the plate on both sides of the hole is modeled and the PWAS transducers operated in pitch-catch mode. A 3-count smoothed voltage tone burst was applied to the T-PWAS and received by the R-PWAS. The presence of the hole in the plate modified the transmitted signal through wave scatter and mode conversion.

Figure 5 shows the comparison between the analytical signal the FEM signal. Figure 6 shows image snapshots of the guided wave pattern in the plate taken at 10 μ s intervals. Two guided wave modes are present, S0 and A0. The A0 mode is considerably slower than the S0 mode. The A0 mode is also much more dispersive than the S0 mode. At $t = 10 \mu$ s, one sees the waves just starting from the T-PWAS. Wave scatter from the hole becomes apparent at $t = 20 \mu$ s, with mode conversion very

clear at $t = 30 \mu\text{s}$. The interaction of the waves with the R-PWAS and the boundaries start to be observable from $t = 40 \mu\text{s}$ onwards. By $t = 80 \mu\text{s}$, most of the wave power has dissipated into the boundaries.

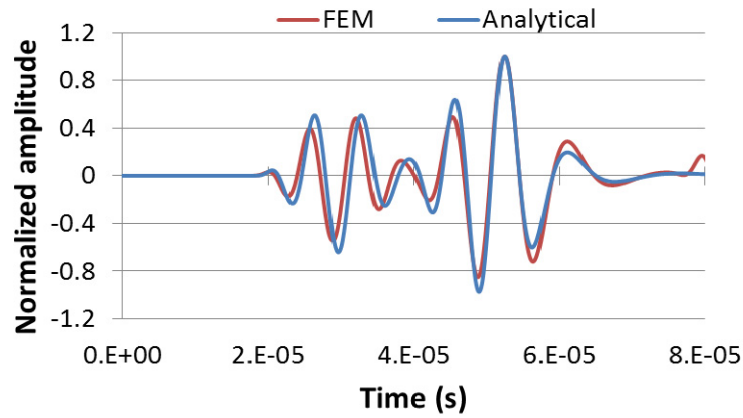


Figure 5: Comparison between the analytical and the FEM receive signal from the R-PWAS.

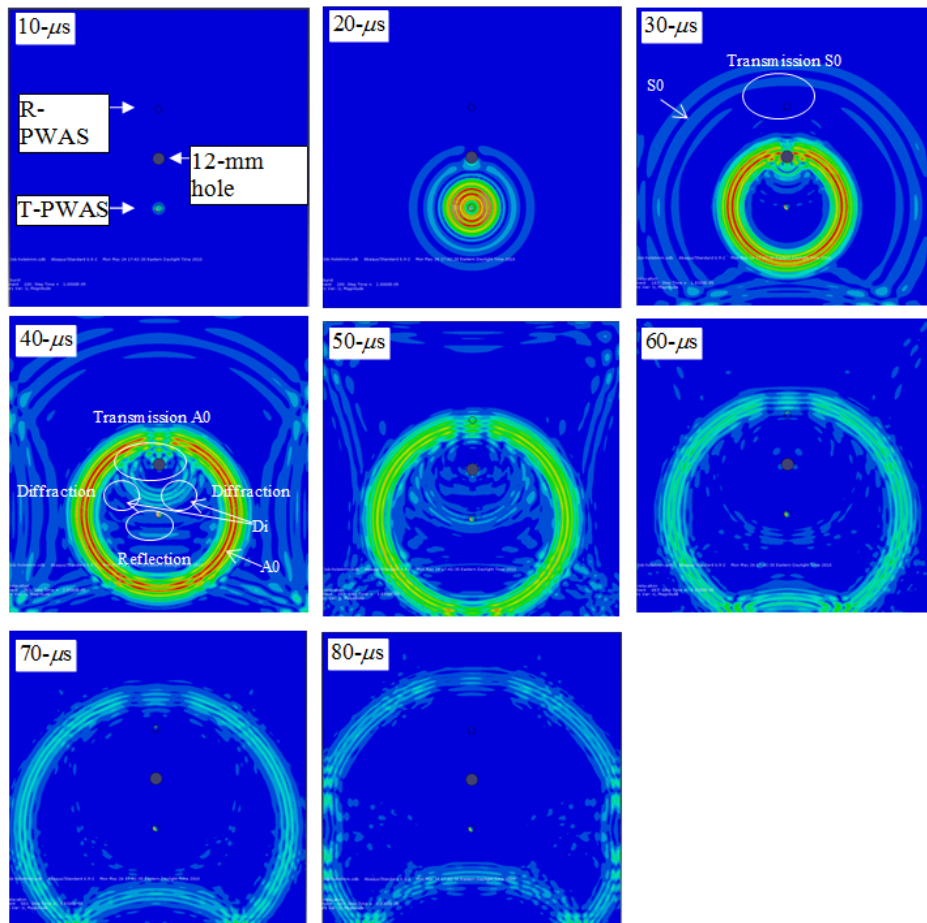


Figure 6: multi-physics finite element method simulations of guided wave generate by a 7-mm PWAS transmitter and scatter from a 12-mm through hole.

CONCLUSION

Theories of guided wave propagation between two PWAS transducers are studied, and analytical model is built to give theoretical waveforms and frequency contents of pitch-catch signal for arbitrary engineering situations. Analytical modeling and finite element modeling have good match with experimental results, and can well describe guided wave propagation between two PWAS transducers. A theoretical explanation for frequency shift in pitch-catch signal is put forward and the theoretical solution matches well with FEM result and experimental data. The experimental frequency shift direction and size are well predicted by the theoretical solution for S0 wave package and FEM result for A0 package.

The analytical model is expected to be extended to 3D circular PWAS analysis, and Bessel function will be studied and included in future work to realize guided wave propagation between two circular PWAS transducers. For further study, the analytical modeling is expected to include damage in the plate structure using a non-linearity aspect.

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