

# Linear and Nonlinear Finite Element Simulation of Wave Propagation through Bolted Lap Joint

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**This paper presents the simulation of wave propagation for health monitoring of a bolted lap joint with linear and nonlinear modeling techniques. The interrogation waves generated by a piezoelectric wafer active sensor (PWAS) propagate in to the structure, interact with the bolted lap joint, carry the lap joint status information with it, and are picked up by a receiver PWAS. In this study, a strip lap joint specimen and a plate lap joint specimen were experimented with guide wave propagation methods. Experimental conditions were then modeled with linear finite element model. The obtained simulation results are not matching the complicated signal from actual experimental signals. A simplified 2-D contact finite element model was used to simulate a strip lap joint specimen. It is found that the nonlinear contact model demonstrates more change at different bolt loading levels.**

## Nomenclature

$d$	=	nominal diameter of the bolt
$F_{clamping}$	=	clamping force
$K$	=	torque coefficient
$T$	=	torque value

## I. Introduction

Evaluation and monitoring lap joint integrity is important for structural health monitoring (SHM). Bolted lap joint is widely used in various applications. In a friction type bolted lap joint, the fasteners create clamping force upon the joint members, and the resulting friction between the contacting surfaces prevents joint slip. Loosen fasteners lead to reduction of clamping force, and compromise the structural integrity. Hence, fastener loading condition evaluation and monitoring need to be addressed for SHM applications. Several researchers proposed various methods to evaluate the bolt loosening conditions in lap joint.<sup>1-4,7-9</sup> These methods sense variation in wave parameters, such as energy, velocity, frequency components, and feature of wave to detect the condition change in bolted lap joint. While these methods can provide valuable information about the bolted lap joint from different aspects, the complex wave propagation conditions through a bolted lap joint make it difficult to fully understand the cause of change in wave patterns.

In this paper, some preliminary results from experimental and numerical study on wave propagation through bolt lap joint are presented. We started with some experimental results. Interesting change patterns were observed in the guided wave propagated through a strip lap joint specimen and a plate lap joint specimen. Two specimens were constructed for the tests: (a) lap joint of two aluminum strips, and (b) lap joint of two aluminum plates. Piezoelectric wafer active sensors (PWAS) transducers were used to generate and receive ultrasonic propagating waves in the specimens. Then, we created numerical models with linear and nonlinear techniques to simulate the experimental conditions. With the linear modeling technique, we were able to reproduce guided wave signal similar to those from other researcher's previous study. With nonlinear technique, some change was observed in the simulated guide wave signal at different bolt loading values. Observations of wave propagation patterns at different fastener loading level are described and discussed in the following sections.

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## II. State of the art

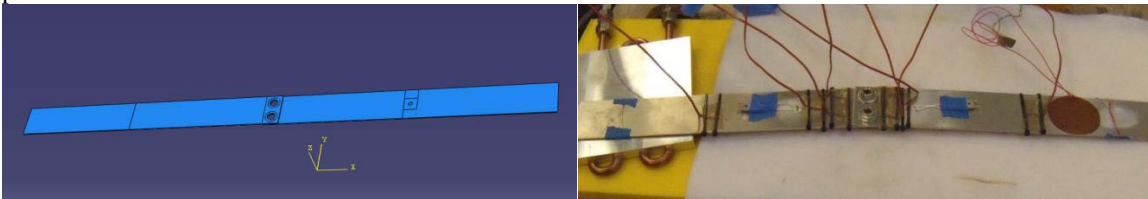
Yang and Chang<sup>1</sup> developed an attenuation-based diagnostic method to detection of bolt loosening in carbon-carbon (C-C) thermal protection system (TPS) panels. The C-C TPS panel was bolted to C/SiC standoff brackets, and the brackets were then bolted to the metallic base structure. PZT (lead zirconate titanate)-embedded sensor washer was developed to create a sensor network. The sensor washers were installed under the head of the bolts connecting the brackets and the base structure. Prototype tests were performed in the laboratory for various loosening conditions of the bolts connecting C-C panel and standoff brackets bolt and the bolts connecting brackets and the base structure. For each combination of panel and bracket loosening, the sensor signals are retrieved and processed into the features energy and SDC, and based on the extracted features the torque levels in the bracket are evaluated collectively to generate two variables: panel torque ( $T_p$ ) and bracket torque ( $T_b$ ). It was found during the verification tests that the attenuation-based diagnostic method was capable of locating a loosened bracket and identifying wither the panel-bracket bolt or the bracket-base structure bolt was loosened.

Zagrai et al.<sup>2</sup> studied lap joint integrity using acousto-elastic method to relate the bolt torque loading with the elastic wave propagation delay. Two 12 in.x2 in.x0.08 in. 2024 aluminum beams were connected by two 3/8-16 3/8-16, grade 8, hex flange, 1 inch steel screws with accompanying 3/8-16 UNC flange nuts to form the lap joint. Piezoelectric transducers (7mm diameter, 0.2 mm thick disk) made with APC 850 piezoceramic material was used to perform pitch-catch experiments. Torque load of 10 ft-lbs to 50 ft-lbs with 10 ft-lbs step were applied to the bolts during the experiments. The wave propagation delay was found to be varying from 0.1  $\mu s$  to 0.6  $\mu s$ . Data analysis showed linear dependence of the arrival time on the applied torque.

Coelho et al.<sup>3</sup> used a classification algorithm based on support vector machines to detect fatigue crack growth, and also to classify the amount of torque in the bold of interest. Clayton et al.<sup>4</sup> explored the feature extraction from guided ultrasonic waves to detect the bolt loosening.

Above mentioned methods sense variation in wave parameters, such as energy, velocity, frequency components, and feature of wave to detect the condition change in bolted lap joint. When wave propagate through lap joint interface, wave transmission, reflection, mode conversion, and damping happen across the lap joint interface. Finite element models are normally created to simulate the bolt joint using contact analysis. However, the contact model requires prior experience for defining the contact pairs and proper calculation parameters such as contact stiffness, friction, etc. Although the correlation between bolt loading value and lap joint clamping force was previously studied by other authors<sup>5,6</sup>, accurate model is not readily available. Hence, thorough experimental and numerical study is necessary to obtain more knowledge about effects of fastener loading on wave propagation through bolted lap joint.

Amerini and Meo studied the structural health monitoring of bolted lap joint using linear and nonlinear ultrasound and vibration approaches such as high-harmonics generation and sidebands modulation<sup>7,8</sup>. They put forward a tightening/loosening state index that indicates the tightening/loosening state of the bolted lap joint. However, for the nonlinear techniques, little theoretical simulation work has been done. Theoretical simulations were attempted using simplified linear finite models.

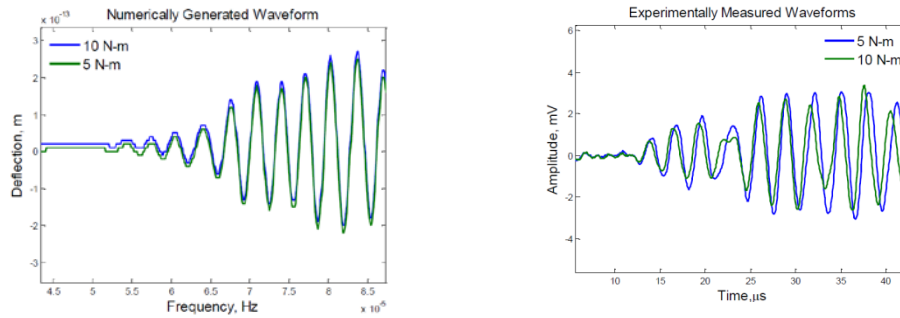


**Figure 1 Modeled and experimental lap joint specimen used by Doyle et al.<sup>9</sup>**

Doyle et al.<sup>9</sup> proposed models using thermal conductance and guided-wave resistance across lap joint interfaces. Finite element package (ABAQUS) was used to model a lap joint specimen. Simplified 3D model of the two lap joint member strips were created with brick elements (C3D8R0). The bolt loading was converted to pressure loading to be applied on partition matching the profile of the bolt washer on the real specimen. The PWAS transmitters were substituted with circular through-all partition on the surface of one plate where the exciting sensor was bonded. Excitation was simulated as a surface traction load where the vector was defined to be radially directed away from the center. The receiving PWAS was not modeled; instead, a data point was assigned at the center of the receiving PWAS. **Figure 1** shows the finite element model and the actual specimen.

In the simulation, 5 Nm and 10 Nm torque loadings were converted to 2000 N/m<sup>2</sup> and 4000 N/m<sup>2</sup> pressure loading on the partition surrounding the bolt hole. It was expected to see the wave slow down as load increases and attenuation as pressure increases. However, analysis on the recorded wave signal from both torque loads did not

show discernible variation between the two loads. **Figure 2** shows the change in waveforms seen in FEM modeling and experimental results.



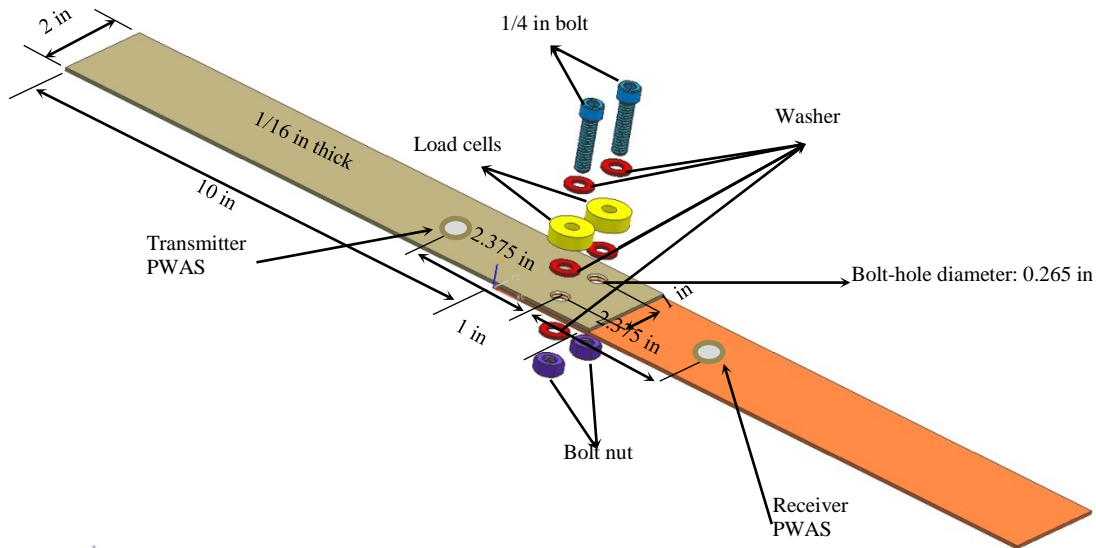
**Figure 2 Experimental and numerical variation in Lamb waves at 5 N-m and 10 N-m, reported by Doyle et al.<sup>9</sup>**

In our work, we duplicated strip lap joint specimen used by previous researcher<sup>9</sup>, and reproduced similar experimental results with guide wave. We found that the strip like specimens used by most investigators (and also by ourselves) do not represent well the actual physical condition encountered in practice, e.g. in the joining of aircraft skins. Our subsequent experiments on large plates have demonstrated that the signals transmitted through the bolted lap joint are much cleaner, especially at higher frequencies where S0 guided waves are predominant.

Numerical models were created to simulate the experimental conditions. We start with a model similar to the model created by Doyle et al.<sup>9</sup>, and obtained similar simulation results. We then improved our model with nonlinear contact element to capture the complicate behavior of lap joint under different bolt loading levels. The results show more changes when different bolt loading values were simulated, and some similarity to the experimetal results were found.

### III. Experiments

We constructed a number of experiments to study the propagation of guided waves through a lap joint with fasteners. Figure 3 presents a typical setup consisting of two strips jointed by two nut-and-bolt fasteners. The load in the fasteners is controlled by the torque applied to the nut-bolt pair. In order to ensure precise application of the load, we also used washer-type load cells ("bolt sensors") inserted under the head of the bolts.



**Figure 3 Geometry and setup of bolted lap joint specimen constructed with two aluminum strips**

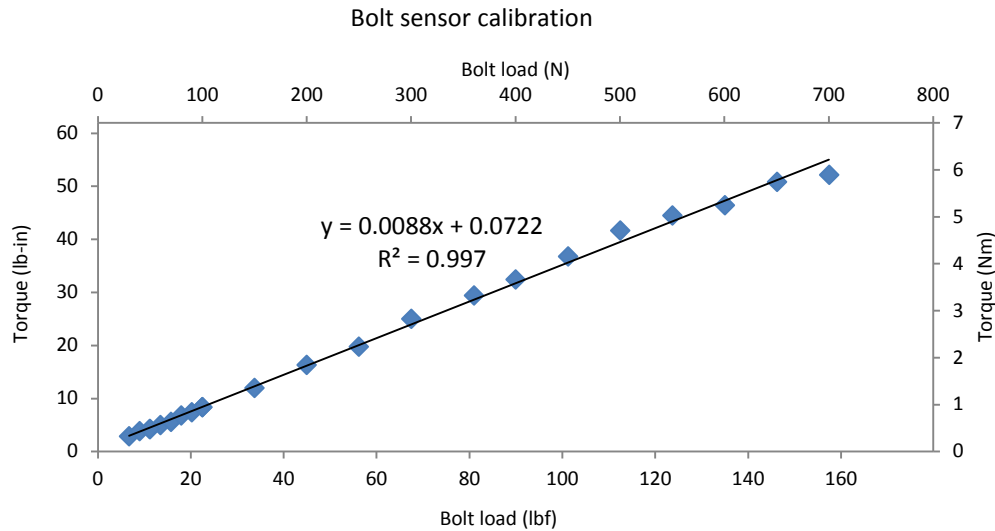
#### A. Strip lap joint specimen

The strip lap joint specimen is constructed with 6061-T6 aluminum strips. Two 11 in. x 2 in. strips of 1/16 in. thick 6061-T6 aluminum strips are used as joint members. Two holes are drilled on each strip for 1/4 in. bolt fasteners. The diameter of the bolt hole is 0.265 in. The two strips have an overlap of 1 in. giving a total surface

contact area of 1 in. x 2 in. The two bolts are placed 1 in. from center to center, symmetric to the center line of the strips. The geometry and setup of the specimen are shown in Figure 3.

A torque wrench (Check-line DTL-100i) is used to apply specific torque to the bolts. The torque range measures the applied torque with a resolution of 0.1 lb-in, and the maximum torque load is rated at 106 lb-in.

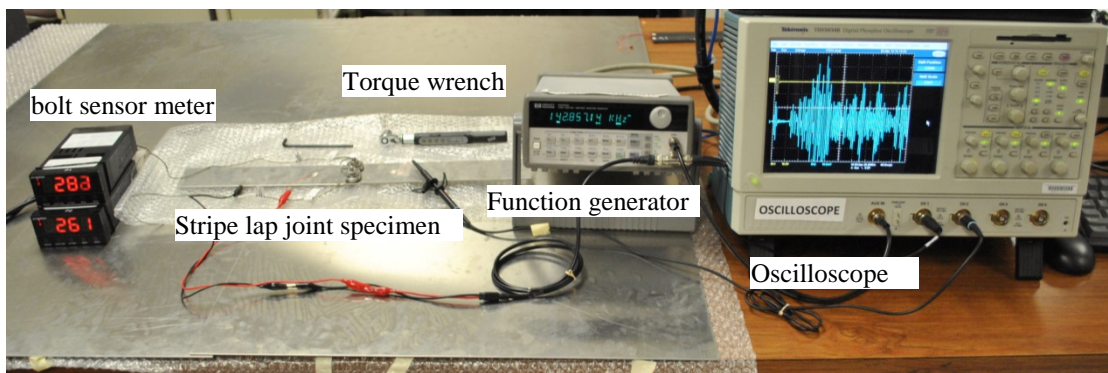
We also use two bolt sensors (Omega LC901) to directly measure the clamping force. The bolt sensors are capable of measuring force up to 2000 lbf, with an accuracy of +/-3.5% full scale output. A calibration experiment was performed separately to evaluate the relation between the applied torque and the actual load in the bolted joint. **Figure 4** shows this relationship which resembles close enough a linear dependency.



**Figure 4** Calibration of the applied torque vs. bolted joint load measured with the bolt sensors.

Two 8.5 mm diameter round PWAS transducers (0.5mm thick, APC 850 piezoceramic) are bonded to the specimen to perform wave propagation tests in pitch-catch mode. Both transmitter and receiver are located along the center line of the specimen, and are 2.375 in. from the bolt line.

The modulated interrogative waves were generated by the transmitter PWAS excited with tone-burst signals of various carrier frequencies. Hanning windowed 3.5-count sine signals generated by an arbitrary function generator with a repetition rate of 10 Hz were used. The signal amplitude was 20 Vpp. During each experiment, a carrier frequency scan was carried out between 100 kHz and 500 kHz with 10 kHz step.



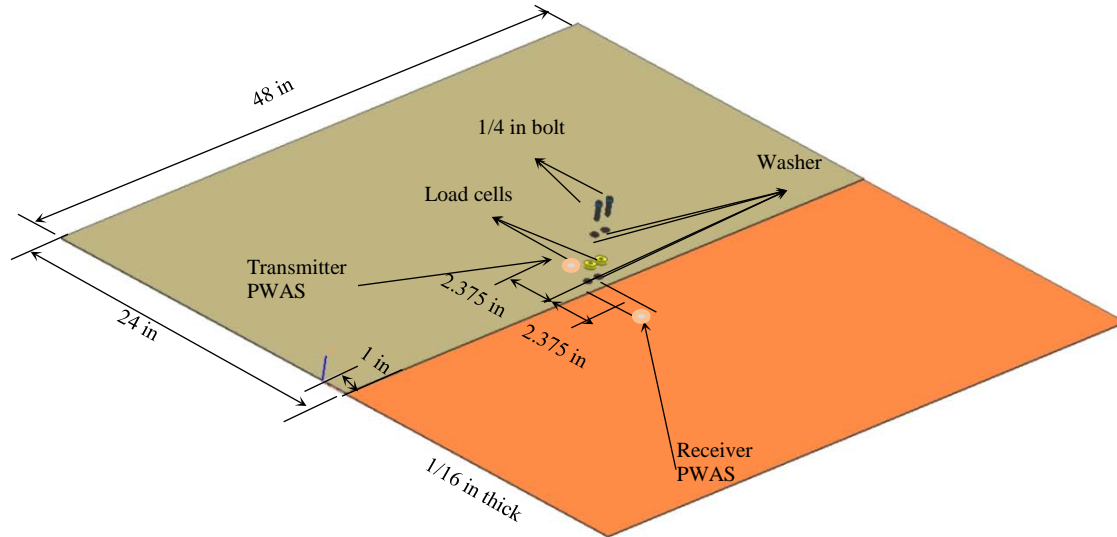
**Figure 5** Strip lap joint specimen experimental setup

The receiver PWAS was connected to a digital oscilloscope for collection of the output voltage signal. The sampling rate was 25 MHz, and total 5000 data points were collected over a 200 μs time interval. **Figure 5** shows the actual experimental setup.

## B. Plate specimen

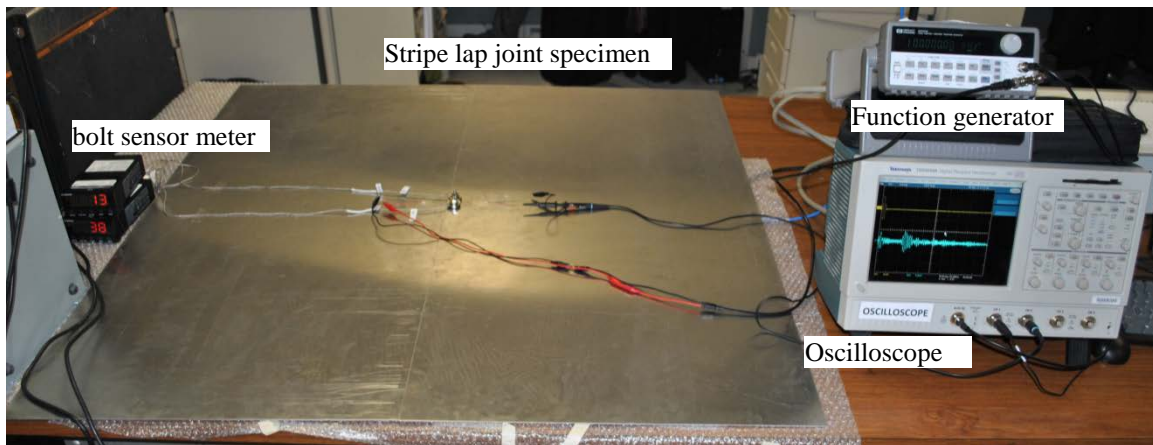
We constructed a second experiment using large plates to reduce the interferences from boundary reflections. In order to keep the two experiments as similar as possible, we instrumented the plate joint with only two fasteners placed identically as in the strip joint. The specimen was constructed with two 24 in.x48 in.x1/16 in. 6061-T6 aluminum plates. The geometry and setup of the specimen are shown in **Figure 6**. The actual experimental setup is shown in **Figure 7**.

One important thing that we observed in our experiments was that the waves from the T-PWAS experienced bouncing off the sides of the strip and, for this reason; the signal received at the other side of the joint was more complicated than expected. As a result, we constructed a second experiment in which we used large plates such that the boundaries are far away from the wave path and the waves bouncing off the boundaries would not interfere with the direct waves traveling through the joint. In order to keep the two experiments as similar as possible, we instrumented the plate joint with only two fasteners placed identically as in the strip joint.



**Figure 6 Geometry and setup of bolted lap joint specimen constructed with two aluminum plates**

The experiments performed on the large plates indicated that the removal of boundary reflections has cleaned up the received signal to a large degree, but not completely. A number of extraneous signals still remained, and we believe that they are due to the scattering of the incoming waves from the bolt holes combined with partial transmission through the joint. In addition, the incoming waves would be reflected at the edge of the first half-plate and sent backwards towards the bolt holes and then scattered forward into the second half plate through the joint. The experimental setup and the results from these two experiments are presented next.

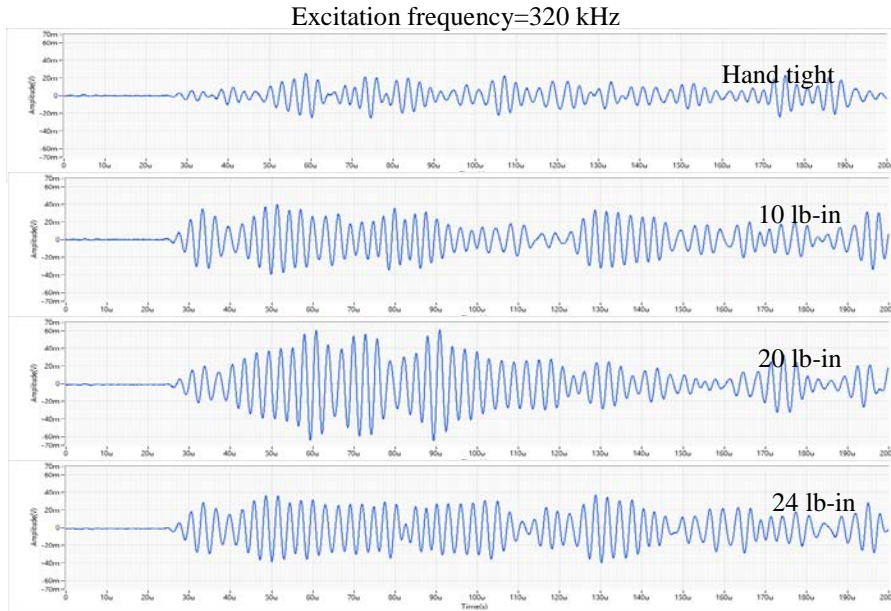


**Figure 7 Plate lap joint specimen experimental setup**

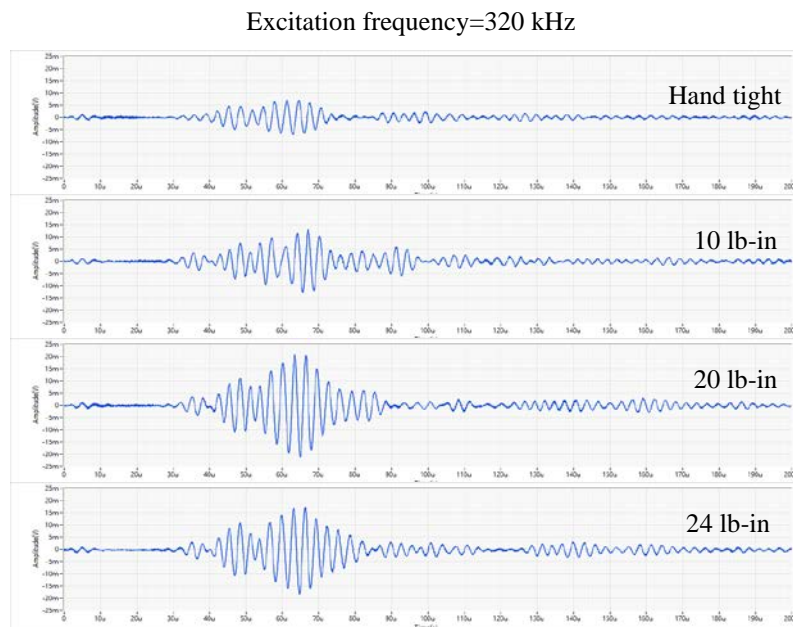
### C. Experimental result



During the strip lap joint experiment, bolt torque loadings of hand tight, 10 lb-in, 20 lb-in, and 24 lb-in were applied consecutively. As an illustration, **Figure 8** presents the receiver PWAS signals at 320 kHz for various bolt torque loading values from hand tight up to 24 lb-in. The signal length is 200  $\mu s$  throughout. One notices that the amplitude of the signal increases with the bolt load up to 20 lb-in; however, the higher load of 24 lb-in resulted in decreased amplitude. At lower bolt loads, we can see clearly wave packets whereas at higher bolt loads, the wave packets start to merge and at the highest bolt load, a continuous signal seems to appear.



**Figure 8 Strip lap joint specimen signal at different bolt torque loading values. Excitation signal frequency is 320 kHz**



**Figure 9 Plate lap joint specimen signal at different bolt torque loadings values. Excitation frequency 320 kHz**

**Figure 9** presents the receiver PWAS signals from plate lap joint specimen, at 320 kHz, for various bolt torque loading values from hand tight up to 24 lb-in. The signal length is 200  $\mu s$  throughout. It's apparent that each wave signals has several high amplitude wave packets that are grouped together, and arrived first. The amplitude of the rest part of the wave signal is much lower than the first arriving wave packet group. In comparison to the strip lap

joint experimental results, these wave signals seem to be much cleaner. Nonetheless, we can see more wave packets when higher torque was applied to the bolts. Similar to the results from strip lap joint experiment, the amplitude of the signal increases with the bolt load up to 20 lb-in, and at higher load of 24 lb-in the amplitude decreases.

It is also very clear that the amplitude of wave signals from plate lap joint experiments is significantly lower than those from the strip lap joint experiments. For example, at 320 kHz frequency and 20 lb-in torque load condition, the maximum signal amplitude from plate lap joint experiment is +/- 60 mV; whereas +/- 20 mV was detected from the strip lap joint experiment. This may be due to the circumferential spreading of the wave energy in the plate following the  $1/r$  law, which is inhibited in the strip by the multiple reflections at the strip boundaries.

#### IV. Finite element simulation

We constructed finite element models using commercial FE software package (ANSYS) to simulate the wave propagation experiments conducted on strip specimen and plate specimen. Multiphysics was used to simulate the pitch-catch method with PWAS transducers. The bolt torque loading values were converted to pressure loading and applied to the elements around the bolt holes. We first simulated the strip specimen with a 3-D linear model; then, the plate specimen was simulated with same technique. After running simulation on these models, we found that the receiver PWAS signals did not match the waveform collected from the experiments. Furthermore, when different pressure was applied, the receiver wave signal did not show much change. This is similar to previous researcher's report. We then created a simplified 2-D model of the strip lap joint specimen with contact element technique. With same simulation parameters, more wave feature was observed in the simulated receiver PWAS wave signal. When the simulated bolt loading was changed, more change was noticed in the receiver PWAS wave signal, and some similarity to the experimental results was noticed.

##### A. Bolt torque conversion method evaluation

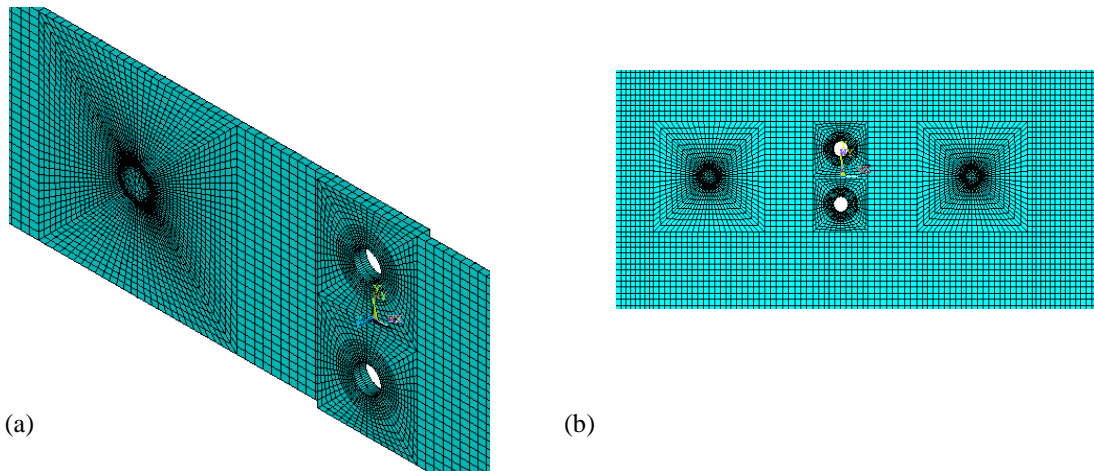
The bolt preload on the lap joint and pressure distribution are very hard to estimate with many uncertainties<sup>11,12</sup>; a widely accepted formula for calculating clamping force is<sup>13</sup>

$$T = KF_{clamping}d \quad (1)$$

Where  $T$  denotes the tightening torque value;  $F_{clamping}$  is the clamping force from the bolt;  $d$  is the nominal diameter of the bolt and  $K$  is the torque coefficient which depends on a variety of parameters including but not limited to geometry and friction of the threads. For the torque coefficient, a value of 0.2 is widely accepted, when the bolt condition is not stated<sup>13</sup>. A uniformly distributed pressure is assumed in our model to be applied on the plate from the washer with the net force equaling to the calculated clamping force.

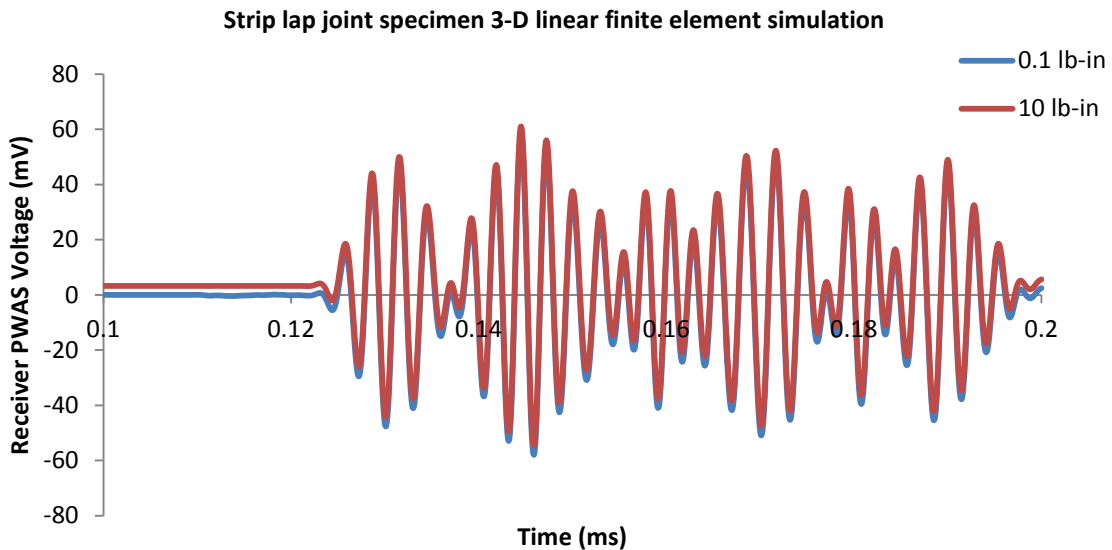
##### B. Linear 3D model and experimental results

3D linear models were created for the strip specimen and plate specimen used in the experiments. Commercial FE software (ANSYS) with multiphysics package was used to create the model. The PWAS was modeled with piezoelectric material type. The transmitter PWAS signals and receiver PWAS signals were simulated with electrode voltage. The dimension of the strip specimen and the plate specimen are shown in Experiments section of this paper. In the model, SOLID45 element was used to create the lap joint members. The bolt hold area is meshed with higher density and created an area to matching the size of the bolt washer. The bolt torque loading was converted to pressure using the conversion formula provided in previous section. The pressure loading was applied to the densely meshed washer area to represent the bolt torque loading.



**Figure 10 linear 3D model details (a) strip lap joint specimen, and (b) plate lap joint specimen**

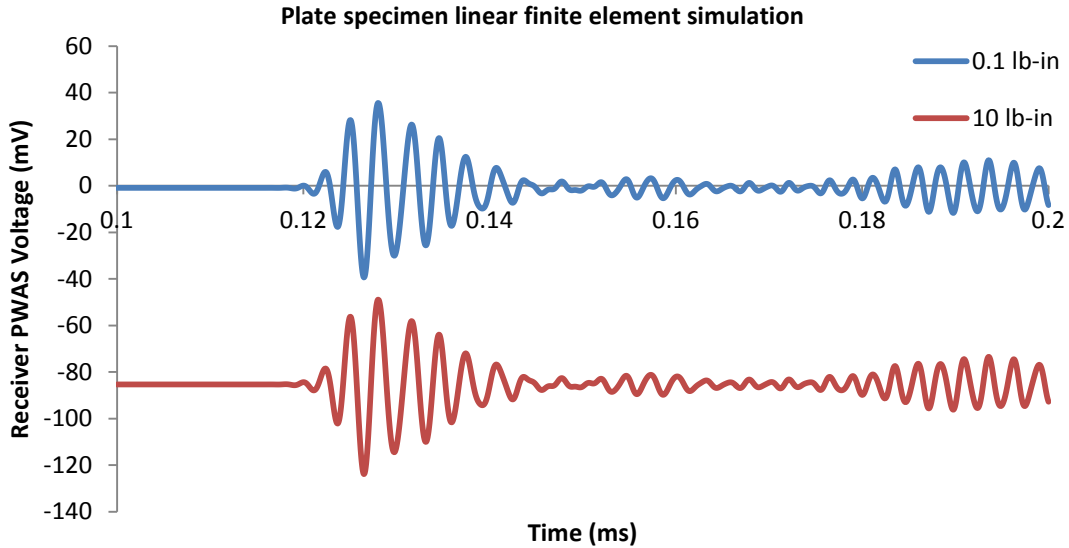
Figure 11 and Figure 12 show the simulation results from the strip lap joint specimen model and plate lap joint specimen model, respectively. Multiple loading levels were simulated during our study; two representative loading levels were plotted for both model simulation cases: 0.1 lb-in and 10 lb-in.



**Figure 11 Simulation results from strip lap joint specimen 3-D linear finite element model at two representative loading levels: 0.1 lb-in and 10 lb-in. The excitation signal is 320 kHz 3.5 count tone burst.**

One can see that the strip lap joint specimen results shows lots of wave packets at the receiver PWAS, while the plate lap joint specimen shows one large initial wave packet and the rest part of the waveform is relatively small. This behavior was previously observed in the experimental resulting waveforms. But if we compare the results from the experiments, it can notice that the experimental results show more complex structures. Most importantly, in the simulation results, although the applied torque changed from 0.1 lb-in to 10 lb-in, the wave signals are almost identical. This is very much different from the experimental results, as shown in **Figure 8** and **Figure 9**.



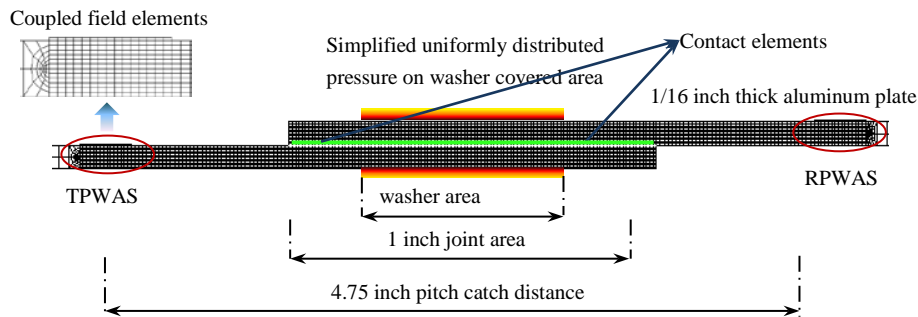


**Figure 12 Simulation results from plate lap joint specimen 3-D linear finite element model at two representative loading levels: 0.1 lb-in and 10 lb-in. The excitation signal is 320 kHz 3.5 count tone burst**  
 The results suggest that the linear model we used in simulating the lap joint specimens are not effectively reflecting the actual wave propagation conditions. This gives us the motivation of a different simulation model.

### C. Nonlinear model design and simulation result

Considering the lap joint construction, we propose to use contact element to model the interface between the two lap joint members. As the first step, we constructed a simplified 2-D model instead of 3-D model.

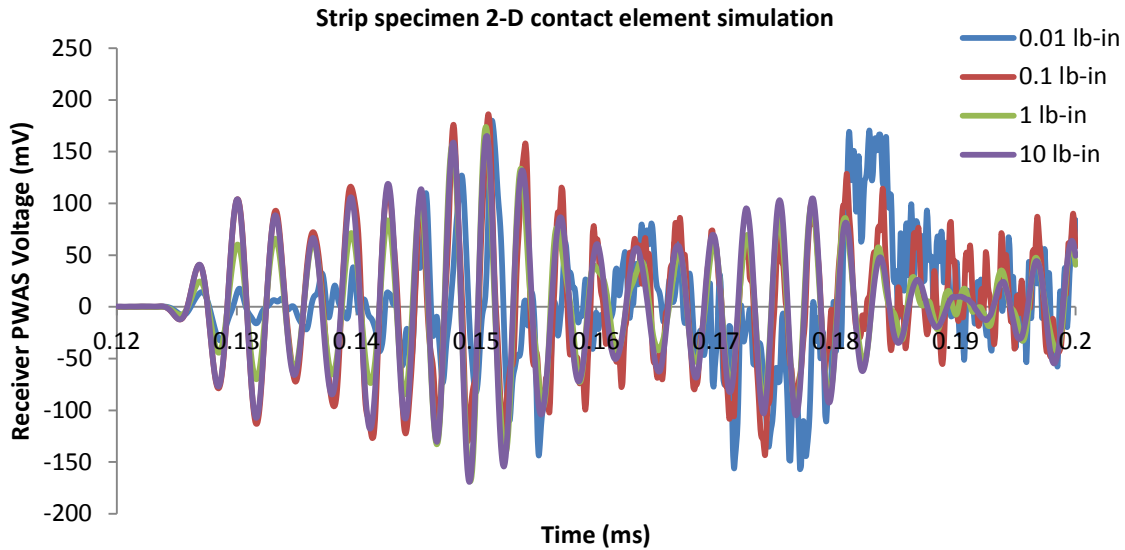
The simplified 2D model of bolted lap joint is shown in Figure 13; the bolt preload is converted to pressure load from the washer.



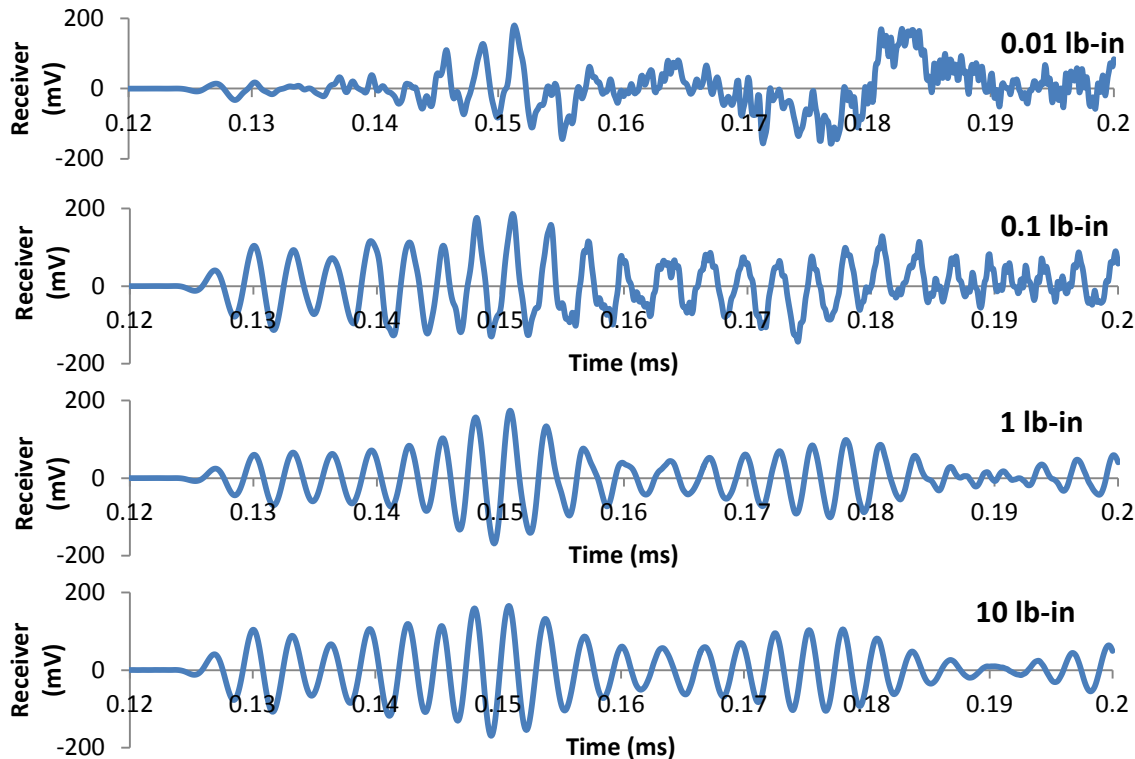
**Figure 13: Simplified 2D finite element model of bolted lap joint**

In a typical transient analysis, when the pressure load is applied on the joint surface, this pressure will propagate in the structure as a pulse wave and interfere with the Lamb waves. To overcome this problem, the transient effect is first turned off, and the pressure is applied on the structure with static analysis performed, then the transient effect is turned on thereafter. Thus, the pressure load applied on the lap joint acts as an initial condition for the transient problem.

Figure 14 shows the simulation results at various bolt loading values. To better observe the signal variation, a smaller 0.01 lb-in loading was introduced to represent the minimal loading condition. One can see from the plot, distinctive changes are presented in the simulated receiver PWAS signal.

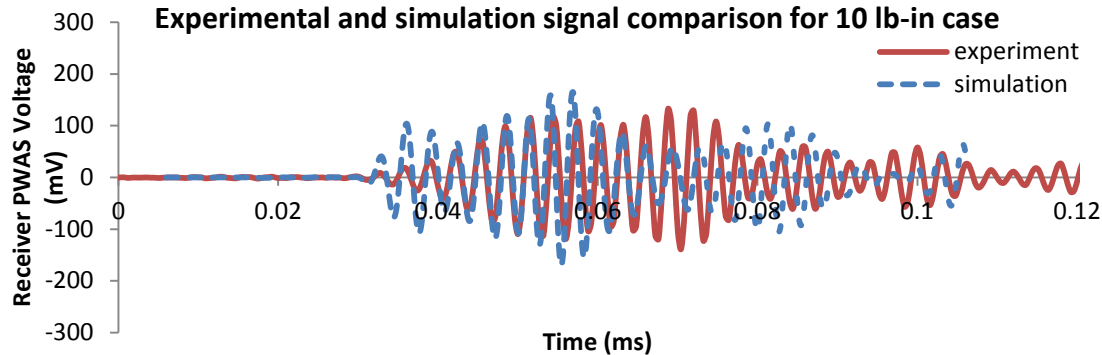


**Figure 14 Simulation results from strip lap joint specimen simplified 2-D nonlinear contact element model at various loading levels between 0.01 lb-in and 10 lb-in. The excitation signal is 320 kHz 3.5 count tone burst**  
 To better review the signals, Figure 15 shows the signals separately. One can notice that when the applied load was changed, the shape and amplitude of the receiver signals show lots of variations. At very low torque loading level of 0.01 lb-in, the signal amplitude is small, and the shape of the signal changed greatly. The transmitted tone burst signal is almost not recognizable.



**Figure 15 comparison of simulation results from simplified 2-D nonlinear contact element model. Four different loading levels are compared, from top to bottom: 0.01 lb-in, 0.1 lb-in, 1 lb-in, and 10 lb-in.**  
 Once the torque loading level was increased to 0.1 lb-in, much better signal was received. The shape of the signal started showing the tone-burst-like form. The amplitude also increased compared with 0.01 lb-in case. increase the torque loading even more, to 1 lb-in, even better tone-burst-like signal start appearing, one can distinguish the different wave packets. This trend continued with the 10 lb-in loading level. The change pattern resembles the

change pattern from experimental results, that is, the amplitude of the signal will increase with the applied bolt loading value in the beginning, and the wave packet seem to vary and separates during the load increasing process.



**Figure 16 Experimental and simulation signal comparison for 10 lb-in case**

If compared with the experimental results as shown in **Figure 8**, for the 10 lb-in loading case, the two signals shows some degree of similarity. A comparison of the signal from simulation and experimental result for 10 lb-in case is shown in Figure 16. One can see the similarity of the two signals. Remarkably, the signal amplitude from simulation and experiment are in close range, which demonstrates the finite element model is much better than the 3-D linear model we used in the previous section.

## V. Conclusion

This paper has presented experimental and numerical simulation results on the propagation of guided waves through a bolted joint under various bolt load values. Piezoelectric wafer active sensor (PWAS) transducers were used for the generation and reception of the guided waves. Two specimen types were used, a strip lap joint and a plate lap joint. The signals measured under various bolt load values and frequency values were studied in order to identify relevant features that change drastically with bolt load. Linear finite element models were used to simulate both the strip lap joint and the plate lap joint specimens. A 2-D nonlinear contact element model was also used to simulate the strip lap joint specimen.

It was found that the interpretation of the bolt load effects on wave transmission through lap joint interface is very challenging, and is insufficiently understood. A review of the state of the art revealed that several other authors have studied this topic without finding a definitive interpretation of the relation between signal changes and bolt load. A detailed finite element model performed by Doyle et al.<sup>9</sup> included in the analysis the local pressure created by the bolt head and washer onto the strips, but could not reproduce the drastic changes observed experimentally in the waveform. Similar results was obtained during this study and confirmed that such linear modeling techniques are not capable of representing the complicated conditions guided wave experiences when propagating through a bolted lap joint. A simplified 2-D nonlinear contact element model was used to simulate the strip lap joint specimen. Although the simulation results from the simplified 2-D nonlinear contact model is still not matching the actual experimental results, much more features were observed in the receiver PWAS signals. The change behavior of the receiver signal shows some similarity to the experimental results.

In studying the wave propagating thought bolted lap joint for SHM applications; it would be of great assistance to have an efficient predictive modeling approach that can capture correctly the effect of bolt load onto the wave transmission through the lap joint. An appropriate numerical model technique is required to simulate the complicated wave propagation interface. Friction, Hertzian contact, nonlinear effects, etc. need to be properly captured and described. These and other aspects will make object of future work.

## Acknowledgments

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## References

- <sup>1</sup>Yang, J., Chang, F.-K.(2006). Detection of bolt loosening in CC composite thermal protection panels: II. Experimental verification. *Struct. Smart Materials and Structures*, 15(2), 591-599.
- <sup>2</sup>Zagrai, A., Doyle, D., and Arritt, B. (2008) Embedded nonlinear ultrasonics for structural health monitoring of satellite joints. *Proc SPIE 6935*.

- <sup>3</sup>Coelho, C. K., Das, S., Chattopadhyay, A., Papandreou-Suppappola, A., and Peralta, P. (2007). Detection of fatigue cracks and torque loss in bolted joints. *Proc. SPIE* 6532.
- <sup>4</sup>Clayton, E. H., Stabb, M. C., Kennel, M. B., Fasel, T. R., Todd, M. D., and Arritt, B. J. (2008) Active ultrasonic joint integrity adjudication for real-time structural health monitoring. *Proc SPIE* 6935.
- <sup>5</sup>Mantelli, M. B. H., Milanez, F. H., Pereira, E. N., and Fletcher, L. S. (2010). Statistical model for pressure distribution of bolted joints. *Journal of Thermophysics and Heat Transfer*, 24(2), 432-437.
- <sup>6</sup>Vand, E. H., Oskouei, R. H., and Chakherlou, T. N. (2008). An Experimental Method for Measuring Clamping Force in Bolted Connections and Effect of Bolt Threads Lubrication on its Value. *Proc. of World Academy of Science: Engineering & Technology*, 48, 457-460 (2008).
- <sup>7</sup>Amerini, F., and Meo, M. (2011). Structural health monitoring of bolted joints using linear and nonlinear acoustic/ultrasound methods. *Structural Health Monitoring*, pp. 659-672.
- <sup>8</sup>Amerini, F., Barbieri, E., Meo, M., and Polimeno, U. (2010) Detecting loosening/tightening of clamped structures using nonlinear vibration techniques. *Smart Mater. Struct.*
- <sup>9</sup>Doyle, D., Reynolds, W., Arritt, B., and Taft, B. (2011). Computational Setup of Structural Health Monitoring for Real-Time Thermal Verification. *SMASIS2011-4991*, 447-453.
- <sup>10</sup>M. Marshall, R. Lewis and R. Joyce (2006). Characterization of contact pressure distribution in bolted joints. *Strain*, pp. 31-43.
- <sup>11</sup>Mantelli, M., Milanez, F. H., Pereira, E. N., and Fletcher, L. S. (2010). Statistical model for pressure distribution of bolted joints. *Journal of Thermophysics and Heat Transfer*, pp. 432-437.
- <sup>12</sup>T. S. group, *Bolt-tightening handbook, Linear Motion & Precision Technologies*, Printed in France, 2001.