CLoVER Transducers for Static and Dynamic Strain Sensing

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ABSTRACT

This paper presents the theoretical formulation and initial experimental investigation of static and dynamic strain sensing using the Composite Long-range Variable-direction Emitting Radar (CLoVER) transducers. CLoVER transducers are manufactured using the piezoelectric fibers integrated with the interdigitated electrodes with wedge-shaped sectors arranged in a circular way. A coupled-field analytical model was developed based on the piezoelectricity constitutive equations with the consideration of the anisotropic material properties of CLoVER transducers. The analytical formulation was derived under a cylindrical coordinate system for a single CLoVER sector. The final sensing formulation contained three strain components and six strain gradients. Sensing signals from nine independent CLoVER sectors with different installation orientations were used to compute for the nine unknown strain related components. Two operation conditions were discussed: (1) static strain sensing and (2) dynamic strain sensing. To validate our analytical model and demonstrate the strain sensing capability of the CLoVER transducers, comparative experiments with conventional strain gauges were conducted on a fix-free aluminum plate. The dynamic strain sensing performance of the CLoVER transducer and the strain gauges were compared. After the calibration, the measurement linearity was examined. The piezoelectric responses from the CLoVER sectors were compared with the analytical prediction.

INTRODUCTION

Composite Long-range Variable-direction Emitting Radar (CLoVER) transducers have been originally developed for directional generation and detection of ultrasonic guided waves for structural health monitoring (SHM) [1]. They are comprised of a circular array of wedge-shaped sectors.

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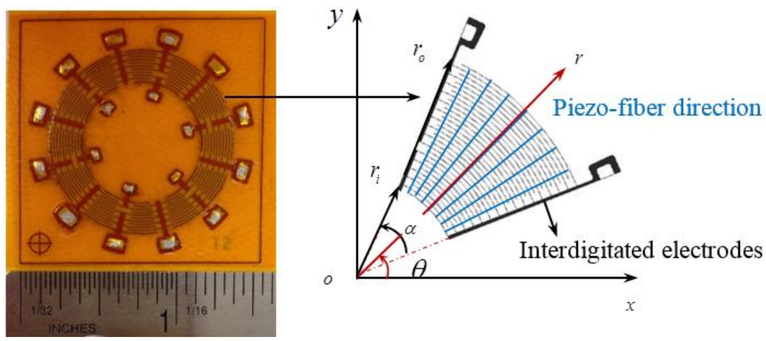

Figure 1: CLoVER transducer and a functional sector with an arbitrary installation angle.

Figure 1 presents the CLoVER transducer with 12 independent sectors manufactured with piezoelectric fibers. Each sector consists of an inner portion and an outer portion. These two parts can work independently or jointly as transmitters and receivers. The piezo-fibers are aligned in the radial direction, while the interdigitated electrodes cover the sectors in the hoop direction.

As a novel multifunctional transducer, CLoVER not only enables the generation and reception of directional ultrasonic guided waves for SHM applications, but also may be used as a strain sensing device for structural usage and load monitoring. Farrar and Lieven addressed the importance of usage monitoring for the damage prognosis in their vision of future SHM systems [2]. It was pointed out that the database of the loading and usage history plays a critical role in the diagnosis of structural fatigue stage and the prognosis of the residual service life. Maley et al. have identified load/strain prediction as an essential aspect for SHM from a U.S. Navy point of view [3]. Previous studies have demonstrated the capability of rectangular piezoelectric Macro-Fiber Composite (MFC) patches for single directional strain sensing [4, 5]. Matt and Lanza di Scalea combined three MFC patches installed in different orientations to form a MFC rosette for acoustic source location in complex structures [6]. Such an application potential is drawing increasing interest to achieve the multifunctional performance of SHM transducers.

This study aims to extend the CLoVER formulation to include the strain sensing capability for structural usage monitoring. What distinguishes the CLoVER transducers from other technologies resides in the following two aspects: (1) CLoVER not only measures the strain components at its center, but also enables to predict the gradients of these stain components; (2) CLoVER has been developed as a multifunctional transducer, which can switch into active sensing mode for damage detection upon the identification of severe operation/loading conditions. This paper starts with the analytical formulation of a single CLoVER sector for static/dynamic strain sensing. Then, the methodology of strain component measurement using multiple sectors is introduced. Experimental case studies are presented. This paper finishes with concluding remarks and suggestions for future work.

CLOVER STRAIN SENSING ANALYTICAL FORMULATION

Consider a single CLoVER sector in an arbitrary orientation θ with respect to the global coordinate system as shown in Figure 1. The CLoVER sector can be modeled as an orthotropic lamina of piezoelectric fibers in a cylindrical coordinate system, with the polling direction along the radial direction [1, 7, 8]. It covers an angular range α, with an inner radius r_i, an outer radius r_o, and a thickness t. Consider the constitutive equation of direct piezoelectric effect

$$D_i = d_{ikl}T_{kl} + e_{ik}^T E_k \tag{1}$$

where D_i is the electrical displacement (charge per unit area); d_{ikl} is the piezoelectric constant, which couples the electrical and mechanical fields; T_{kl} is the mechanical stress; e_{ik}^T is the dielectric permittivity measured at zero mechanical stress ($T=0$); E_k is the electrical field. Casting Eq. (1) in matrix form and expanding the expression under the cylindrical coordinate system yields

$$D_1 = \begin{bmatrix} d_{11} & d_{12} & 0 \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \left(\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix} - E_1 \begin{Bmatrix} d_{11} \\ d_{12} \\ 0 \end{Bmatrix} \right) + e_{11}E_1 \tag{2}$$

The stiffness matrix $[Q]$ can be calculated from the elastic modulus of the piezo fibers. The subscript numbers adhere the nomenclature for laminate composites. In the cylindrical coordinate system, they coincide with the radial and hoop directions. It should be noted that, in Eq. (2), the mechanical-induced strain equals the total strain ($\varepsilon_{11}, \varepsilon_{22}, \gamma_{12}$) minus the piezoelectric-induced strain. Solving for the electric displacement D_1 yields

$$D_1 = a_1\varepsilon_{11} + a_2\varepsilon_{22} + a_3E_1 \tag{3}$$

where

$$a_1 \equiv d_{11}Q_{11} + d_{12}Q_{12}; \quad a_2 \equiv d_{11}Q_{12} + d_{12}Q_{22}; \quad a_3 \equiv e_{11} - d_{11}^2Q_{11} - 2d_{11}d_{12}Q_{12} - d_{12}^2Q_{22} \tag{4}$$

It should be noted that the strain components $\varepsilon_{11}, \varepsilon_{22}, \gamma_{12}$ in the cylindrical coordinate are expressed by the strain components $\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy}$ in the global Cartesian coordinate through coordinate transformation. In this formulation, a non-uniform strain field is considered. Since the CLoVER area is small, we assume that the strain gradients are constants under the sectors and the strain field can be expressed by the stain components ε_{xx}^o, ε_{yy}^o, γ_{xy}^o at the CLoVER center and the strain gradients through a spatial Taylor expansion.

In this derivation, the influence from the measuring instrument is considered. Figure 2 shows the equivalent measuring electronic circuits for the static and dynamic strain sensing cases. For static strain sensing, the measuring instrument is treated as a capacitor with a capacitance of C_e. For dynamic strain sensing, the measuring instrument is treated as a resister with an impedance of Z_e and admittance of Y_e ($Y_e = 1/Z_e$) [9]. Figure 2c shows the interdigitated electrodes and the electric field within a CLoVER sector.

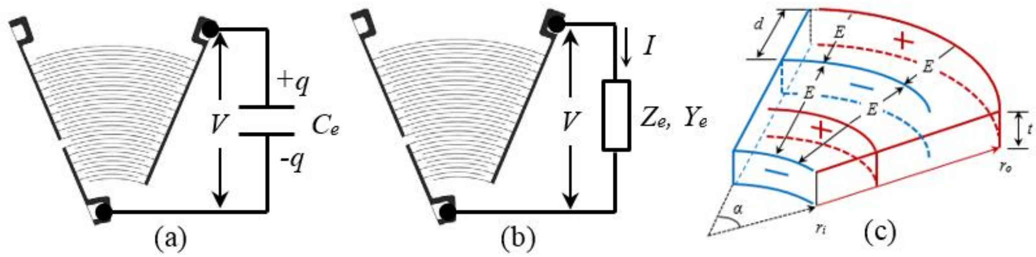

Figure 2: Equivalent measuring electronic circuits: (a) static strain sensing; (b) dynamic strain sensing; (c) interdigitated electrodes and electric field of a CLoVER sector.

Static Strain Sensing Formulation of a Single CLoVER Sector

Recall the relation between electric field and voltage

$$V = -\frac{\iiint E_1 dxdydz}{A} \tag{5}$$

where A is the effective electrode area shown in Figure 2c. It can be estimated by $A = \alpha t\left(r_o^2 - r_i^2\right)/2d$, where d is the distance between the neighboring electrodes; t is the thickness of the sectors. The electric field E_1 in Eq. (5) can be solved from Eq. (3). Conducting the integration of the electric displacement D_1 over the sector volume yields

$$\int_0^t \int_{r_i}^{r_o} \int_{\theta-\frac{\alpha}{2}}^{\theta+\frac{\alpha}{2}} D_1 d\theta dr dz = \int D_1 dV = \int_0^d \int D_1 dA dr = C_e V d \tag{6}$$

Substituting Eq. (6) into Eq. (5), one arrives at the final sensing voltage of a CLoVER sector, which is a function of the strain components at the center, the strain gradients, the installation angle, and the sector dimensions.

$$V(\theta) = \frac{t(r_o - r_i)}{(Aa_3 + C_e d)} \left\{ \begin{array}{l} \varepsilon_{xx}^o XX(\theta) + \frac{\partial \varepsilon_{xx}}{\partial x} XX_1(\theta) + \frac{\partial \varepsilon_{xx}}{\partial y} XX_2(\theta) \\ +\varepsilon_{yy}^o YY(\theta) + \frac{\partial \varepsilon_{yy}}{\partial x} YY_1(\theta) + \frac{\partial \varepsilon_{yy}}{\partial y} YY_2(\theta) \\ +\gamma_{xy}^o XY(\theta) + \frac{\partial \gamma_{xy}}{\partial x} XY_1(\theta) + \frac{\partial \gamma_{xy}}{\partial y} XY_2(\theta) \end{array} \right\} \tag{7}$$

where $XX(\theta)$, $YY(\theta)$, $XY(\theta)$ etc. are the explicit coefficients containing the installation angle and CLoVER dimensions calculated from the integration.

Dynamic Strain Sensing Formulation of a Single CLoVER Sector

For dynamic measurement, we assume a measuring instrument of input impedance Z_e and admittance Y_e ($Y_e = 1/Z_e$). The CLoVER sector dynamics is ignored, i.e., it is assumed that the operational frequencies are well below the resonance frequency of the sensing sector (e.g., hundreds of Hz to low kHz operational frequencies). The dynamic sensing equation is derived from the time derivative of Eq. (5).

$$\dot{V} = -\frac{\iiint \dot{E}_1 dxdydz}{A} \tag{8}$$

Conducting the volume integration of the electric displacement rate $\dot{D}_1$ yields

$$\int_0^t \int_{r_i}^{r_o} \int_{\theta-\frac{\alpha}{2}}^{\theta+\frac{\alpha}{2}} \dot{D}_1 d\theta drdz = \int \dot{D}_1 dV = \int_0^d \int \dot{D}_1 dAdr = Y_e Vd \tag{9}$$

Consider a harmonic solution, the dynamic response and the strain field have the following time derivative relations

$$\dot{V}(t) = i\omega\hat{V}e^{i\omega t} = i\omega V(t); \quad \dot{\varepsilon}(t) = i\omega\hat{\varepsilon}e^{i\omega t} = i\omega\varepsilon(t) \tag{10}$$

where i is the imaginary unit; ω is the angular frequency. For a lossless measuring instrument

$$Y_e = i\omega C_e \tag{11}$$

Combine Eqs. (9), (10), and (11) to solve Eq. (8) yields the sensing formulation, which one finds the same solution as the static case given in Eq. (7). It should be noted that practical measuring instruments do not have a purely capacitive input admittance, since some resistive losses and inductive coupling are always present; similarly actual CLoVER transducers always have internal dissipation, especially at high frequencies. For these reasons, the simplification using Eq. (11) may not be always possible. It is also noticed that, in practice, the static measuring voltage will decay due to the resistive losses within the electronic circuits.

STRAIN MEASUREMENTS USING MULTIPLE CLOVER SECTORS

Recall the static and dynamic strain sensing equations of a single CLoVER sector in Eq. (7). For a specific design, the sector coverage angle α is a constant. For a specific installation orientation with respect to the global Cartesian coordinate system, θ is also a constant. For a measurement $V(\theta)$, the only unknowns are the target strain components and their gradients. Given nine CLoVER sectors with different installation orientations, we can arrive at nine equations of sensing voltage with nine unknowns. Thus, we can solve for the target strains and strain gradients. Since CLoVER may have more than nine sectors, we can use all the sectors and arrive at an overdetermined equation system. The equations of the measurements for various orientations can be casted into matrix form as

$$\hat{\boldsymbol{V}} = p\boldsymbol{X}\hat{\boldsymbol{\varepsilon}} \tag{12}$$

where

$$\hat{\boldsymbol{V}} = \left[\hat{V}(\theta_1) \quad \cdots \quad \hat{V}(\theta_9)\right]^T; \quad p = \frac{\kappa t(r_o - r_i)}{(Aa_3 + C_e d)} \tag{13}$$

$$\boldsymbol{X} = \begin{bmatrix} XX(\theta_1) & XX_1(\theta_1) & XX_2(\theta_1) & \cdots & XY(\theta_1) & XY_1(\theta_1) & XY_2(\theta_1) \\ \vdots & & & & & & \vdots \\ XX(\theta_9) & XX_1(\theta_9) & XX_2(\theta_9) & \cdots & XY(\theta_9) & XY_1(\theta_9) & XY_2(\theta_9) \end{bmatrix} \tag{14}$$

$$\hat{\boldsymbol{\varepsilon}} = \begin{bmatrix} \hat{\varepsilon}_{xx}^o & \dfrac{\partial \hat{\varepsilon}_{xx}}{\partial x} & \dfrac{\partial \hat{\varepsilon}_{xx}}{\partial y} & \cdots & \hat{\gamma}_{xy}^o & \dfrac{\partial \hat{\gamma}_{xy}}{\partial x} & \dfrac{\partial \hat{\gamma}_{xy}}{\partial y} \end{bmatrix}^T \tag{15}$$

where κ is the calibration factor. Thus, the strain can be estimated by Eq. (16). For dynamic strain sensing, $\hat{\varepsilon}$ becomes the harmonic strain amplitude and $\hat{V}$ is the harmonic voltage amplitude.

$$\hat{\boldsymbol{\varepsilon}} = \frac{1}{p}\boldsymbol{X}^{-1}\hat{\boldsymbol{V}} \tag{16}$$

EXPERIMENTS FOR CLOVER STRAIN SENSING

Figure 3 presents the experimental setup for CLoVER strain sensing tests. A 32-in long, 17-in wide, and 0.128-in thick aluminum plate was used as the test structure with a fix-free boundary condition. A 12-sector CLoVER transducer, with an inner radius of 10 mm and an outer radius of 15 mm, was surface-bonded on the center top of the plate. A strain gauge rosette was installed on the center bottom of the plate. A fourth strain gauge was installed 10 mm away from the rosette and closer to the fixed end for the calculation of strain gradient. A quarter bridge circuit and a post-amplifier were used to obtain the strain gauge readings, while the CLoVER sectors were directly connected to the oscilloscope. Initial deformations were introduced at the plate free end and were rapidly removed to trigger free vibrations of the plate. The readings from the strain gauges and the CLoVER sectors were recorded.

Figure 4 presents the comparison of sensing signals between a strain gauge and a CLoVER sector. The CLoVER sector signal shows good performance in monitoring the structural vibration. Its advantage over the conventional strain gauge can be noticed in its sensitivity for high frequency vibration components. The conventional strain gauge, on the other hand, does not exhibit satisfying performance at higher frequencies.

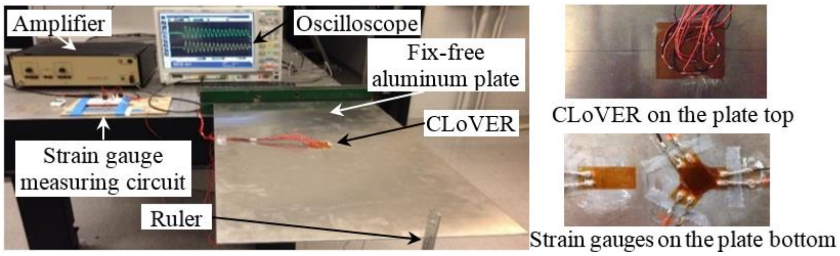

Figure 3: Experiment setup for CLoVER strain sensing.

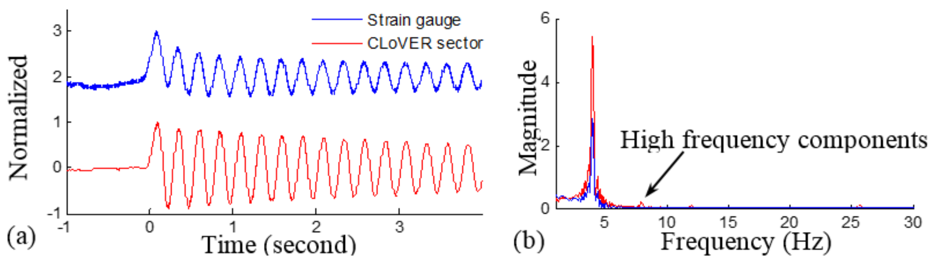

Figure 4: Comparison of sensing signals between a strain gauge and a CLoVER sector: (a) time domain signal; (b) frequency domain signal.

Figure 5a shows the sensing response linearity test of the 180 degree CLoVER sector under various plate free end deformations. It can be noticed that the CLoVER sector presents good response linearity, which means the measurements are accurate after a calibration with the strain gauge at an arbitrary deformation within the tested range.

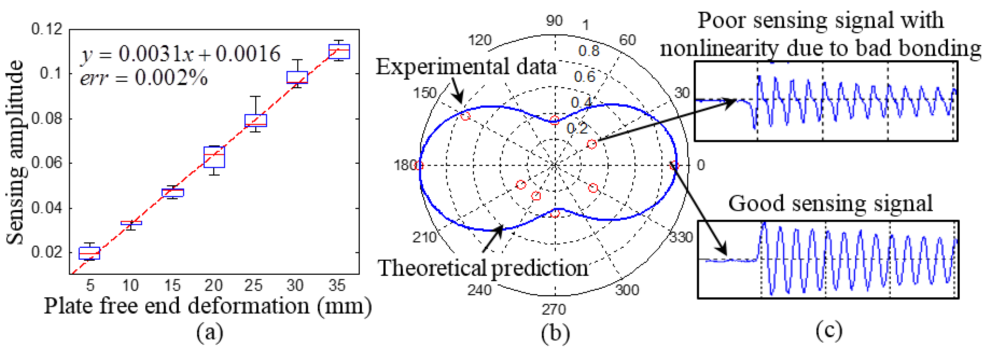

Figure 5: (a) Linearity test of sensing signals; (b) comparison between theoretical responses with experimental data; (c) poor sensing signal due to bad bonding.

Figure 5b presents the comparison between the theoretical responses (after calibration) with experimental measurements. Many of the experimental data points agree well with the theoretical prediction, however several sectors did not match the prediction. Figure 5c shows the sensing signals of two sectors. The signal from the 0 degree sector, which demonstrates good agreement with theoretical prediction, shows clear free vibration pattern. However, the signal from the 30 degree sector, which deviates from the prediction, shows nonlinear response due to bad bonding of the sector.

CONCLUDING REMARKS

In this paper, we presented the theoretical formulation and initial experimental investigation of using CLoVER transducers for strain sensing. The formulation was derived using the piezoelectric constitutive equations, treating CLoVER sectors as orthotropic composite laminate in a cylindrical coordinate system. The methodology of strain measurement using multiple CLoVER sectors was proposed. The theoretical study showed that CLoVER not only enables the monitoring of strain components, but also allows the estimation of strain gradients. The initial experimental investigation demonstrated potential of CLoVER transducers for strain measurements. Further experimental investigations are still required to validate the CLoVER measurements.

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